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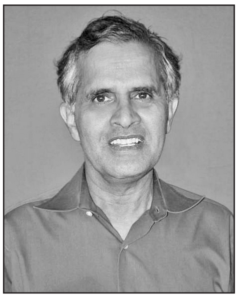
Editorial

Dr K. Balaji Rao, completed his M.E and Ph D from Indian Institute of Science, Bangalore, in the years 1984 and 1990 respectively. After a very brief stint at Tata Consulting Engineers, Bengaluru, he joined CSIR-Structural Engineering Research Centre, Taramani, Chennai in the year 1990. Currently he is a chief scientist at CSIR-SERC and is a faculty of AcSIR. His research interests include stochastic mechanics, stochastic modelling of natural hazards, Markov chain modelling of engineering systems at different scales, vulnerability analysis of built environment and risk-consistent design of engineering structures. He has led a number of in-house and sponsored R&D projects. Important among the sponsored projects are: (i) A methodology for risk informed in-service inspection for safety related systems, (ii) Seismic vulnerability analysis of brick masonry buildings, (iii) Development of probabilistic seismic hazard map of India. He has published a number of refereed journal and conference papers apart from number of R&D reports. He was deputed by CSIR-SERC to Rice University and University of Notre Dame in 1994 under SERC-UNDP fellowship. He did collaborative work with Prof. R. Rackwitz. Universität der Bundeswehr München, Germany, in 1999, when he visited CSIR-SERC under CSIR-DAAD exchange programme. He was a visiting scientist at Electronic Enterprises Laboratory, Department of Computer Science and Automation, IISc, Bangalore, in 2006. He has guided two Ph D candidates and a number of B.Tech/M.E students. He is a member of Editorial board of the following journals: Journal of Structural Engineering, CSIR-SERC, International Journal of Engineering under uncertainty: hazards, assessment and mitigation, Open Journal of Safety Science and Technology, Journal of Chemical Engineering and Materials Science. He has been identified as a mentor at CSIR-SERC and is currently working in the area of stochastic multi-scale modelling and sustainability-based design.



Dr. Chandrasekhar Putcha is a Professor in the Department of Civil and Environmental Engineering at California State University, Fullerton. He has been at this place since 1981. Before that he was on the research faculty at West Virginia University, Morgantown, WV and a post-doctoral fellow at University of Sherbrooke in Canada. His research areas of interest are –Reliability, Risk Analysis, Optimization and Mathematical Modeling. Because of his interdisciplinary areas of research, Dr. Putcha has published more than 135 research papers in various disciplines such as Engineering, Medicine, Kinesiology, Political Science and Sociology. He has done consulting work for several leading companies and received research grants from companies such as Boeing, Northrop Grumman Corporation (NGC) and from federal agencies such as – NASA, Navy, Air Force, US Army Corps of Engineers.

Dr. Putcha did his Ph.D.and M.S. in Civil Engineering from Indian Institute of Technology, Kanpur in 1975 and 1971 respectively and B.S. from IIIT/BHU in 1969. He was the Head of Civil and Environmental Engineering Department at California State University, Fullerton (CSUF) from 1996-2002.

Key Awards

1. CSUF Outstanding Professor (2006-07)
   (First time recipient from the college of Engineering in 44 years Since this award was instituted in 1963)
2. College of Engineering and Computer Science outstanding Professor award (May 1994)
3. "Prestigious Engineering Educator" Award from Orange County Engineering Council (OCEC), Feb. 26, 2005.
4. "Distinguished Engineering Educator" Award from Orange County Engineering Council (OCEC), Feb. 24, 2001.

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Generation of site-specific response spectra through fuzzy-stochastic modelling

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Abstract

The acceleration response spectra are usually used for specifying the seismic ground motions for design. Two of the major factors to be considered while determining the response spectra for a given site are the variability in ground motions expected at the site and the local site conditions. Probabilistic approaches have been used internationally to represent the stochastic variations in ground motion at the site. However, the site conditions are usually defined in a general and qualitative manner in linguistic terms (viz., hard rock, stiff soil). This gives rise to uncertainties, which can best be modelled by using the theory of fuzzy sets. A methodology for generating acceleration response spectrum by using fuzzy-stochastic models of earthquake ground motions is proposed in this paper. The usefulness of the proposed methodology in developing site-specific acceleration response spectra is illustrated through an example problem. From the results obtained, it is noted that proper classification of soil sites is important for design, indicating the need for seismic microzonation.

Keywords- Seismic ground motion, response spectrum, fuzzy stochastic modelling

1. Introduction

Ground vibrations during an earthquake can cause severe damage to structures leading to loss of human lives and property. The ground vibrations at a site are influenced by various factors, the most important of which are (Newmark and Rosenblueth, 1971): i) Earthquake mechanism, ii) Properties of the medium of the path of propagation of the seismic waves, and iii) Local site conditions. It has long been realised that the presence of soft soil layers near the earth's surface causes an increase in the amplitudes of seismic waves. This phenomenon is known as site amplification, and is mainly caused due to the low impedance of soil layers near the earth's surface (Safak, 2001). The magnitude of site amplification depends upon the depth to the bed rock as well as the type, thickness and properties of the soil layers above the bed rock. Hence, these factors need to be taken into consideration while determining the earthquake ground motions at a given site.

The earthquake excitations are normally characterised using the earthquake response spectrum for the design purposes. The response spectrum used in design should take into account the site geology. This requires a thorough classification of soil sites. It is observed that the soil conditions are specified in a general and qualitative manner using linguistic terms in the codes of practice. Also, there will be variations in soil properties from point to point in a given site. These variations should be taken into account while developing the acceleration response spectrum for a given site. A methodology for determining the acceleration response spectrum for a given site using fuzzy-stochastic models of earthquake excitation is presented in this paper. In the proposed methodology, the uncertainties in the earthquake ground motion expected at a site arising due to the use of linguistic terms for defining site conditions and the stochastic variations in ground motion are taken into consideration by representing the ground motions using a fuzzy-stochastic model. The usefulness of the methodology in developing site-specific acceleration response spectra is illustrated through an example problem.

2. Methodology for Determining Acceleration Response Spectrum Using Fuzzy-Stochastic Modelling

The acceleration response spectra are usually used for specifying the seismic motions for design. One of the major factors to be considered while determining the response spectra for a given site is the variability in ground motions expected at the site. The uncertainties in the earthquake ground motion expected at a site are due to: i) the use of linguistic terms for defining soil conditions, and ii) the stochastic variations in ground motion at the site. The uncertainties associated with the linguistic terms can be handled more rationally by using the theory of fuzzy sets. Hence, a hybrid approach, which can take into account both the probabilistic uncertainties and the fuzzy uncertainties, is required for rationally determining the response spectra.

In the proposed methodology, a fuzzy-stochastic model is used for representing the earthquake ground motions at the site. The severity of ground motion at a site is often represented by PGA. For a given earthquake magnitude, the PGA at a site depends mainly upon the distance of the site from the source, lithological and tectonic features between the source and the site, and the soil conditions at the site (Newmark and Rosenblueth, 1971). Since the soil conditions at a site are normally expressed using linguistic terms (such as soft soil, hard rock), it is more appropriate to represent the PGA with a fuzzy set. Hence, in this study, the PGA at the site has been represented using a fuzzy set. To take into account the stochastic variations in earthquake ground motions, 100 accelerograms for each possible realisation of the PGA has been generated.

2.1 Proposed Methodology

For determining the acceleration response spectra for a given site, it is important to classify the soil conditions at the site. The classification given in IBC 2000 (NEHRP soil profiles) is used in the present study (see Table 1). The proposed procedure for determining the acceleration response spectrum is given below (Anoop *et al.*, 2002).

1. Determination of characteristic earthquake magnitude for each seismic zone: The characteristic magnitude of earthquake for the seismic zone under consideration is determined based on the data available on previous earthquakes in that zone using Bootstrap method (Balaji Rao *et al.*, 1999, 2003).
2. Determine the peak ground acceleration (PGA) on rock corresponding to the maximum earthquake magnitude using a suitable attenuation relation. The determined PGA is represented as a random variable and is converted into an equivalent fuzzy variable to represent the uncertainties in its value.
3. Generation of accelerograms: Corresponding to the values at different λ-cut levels of the fuzzy set for PGA, generate different accelerograms (ensemble of 100 accelerograms corresponding to each PGA value of a λ-cut).
4. Generation of fuzzy acceleration response spectrum on rock (Site Class B): Using the 100 simulated accelerograms corresponding to the PGA value of a given λ-cut level, determine the acceleration response spectrum. The outer envelope of the generated acceleration response spectrums is chosen as the acceleration response spectrum corresponding to that particular λ-cut level. In this manner, the fuzzy acceleration response spectrum for a particular site in a particular seismic zone can be formulated.
5. Generation of fuzzy acceleration response spectrum on other soil profiles: Fuzzy acceleration response spectra for other soils are determined by multiplying the response spectrum on rock with the short period (0.1-0.5s) and mid-period (0.5-2.0s) amplification factors given in IBC 2000 (see Tables 2 and 3).

The following assumptions are made.

1. The duration of the earthquake is assumed to be the same for the different λ-cut levels of the fuzzy set for PGA.
2. It is assumed that the short- and mid- period amplification factors given in IBC 2000 are applicable for periods < 0.1s and for periods > 2.0s, respectively.

An example problem is given in the next section to illustrate the proposed methodology.

2.2 Example

A firm ground at a focal distance of 15 Km from an earthquake source is considered. Using the Bootstrap method for determining the confidence intervals for earthquake magnitude (Balaji Rao *et al.*, 1999, 2003), the characteristic magnitude for a given region is determined as 7 in Richter scale. For a magnitude of 7,

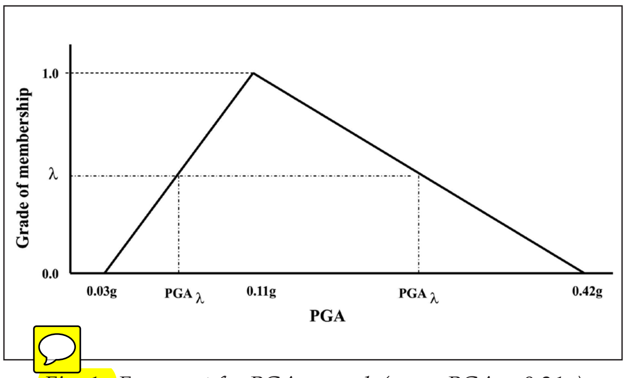

Fig. 1 : Fuzzy set for PGA on rock (mean PGA = 0.21g)

Table 1. NEHRP Soil Profile Types

Site Class	Description	$\bar{v}_s$, N or N_{ch}, $\bar{s}_u$
A	Hard Rock	$\bar{v}_s$ > 1500 m/s
B	Rock	760 m/s < $\bar{v}_s$ ≤ 1500 m/s
C	Very dense soil and soft rock	360 m/s < $\bar{v}_s$ ≤ 760 m/s or N > 50 or $\bar{s}_u$ > 100 kpa
D	Stiff soil	180 m/s ≤ $\bar{v}_s$ ≤ 360 m/s or 15 ≤ N ≤ 50 or 50 kpa ≤ $\bar{s}_u$ ≤ 100 kpa
E	Soft soil	$\bar{v}_s$ < 180 m/s or with N < 15, $\bar{s}_u$ < 50 kpa or any profile with more than 3 m of soft clay defined as soil with PI > 20, w ≥ 40 %, and $\bar{s}_u$ < 25 kpa
F	Soils requiring site specific evaluation	

Note: $\bar{v}_s$ - average shear wave velocity for the top 30 m of the soil; N, N_{ch} – average standard penetration resistance values for the top 30 m of soil; $\bar{s}_u$ - average undrained shear strength of soil for the top 30 m; PI – plasticity index; w – moisture content in percent

Table 2. Values of F_a as a Function of Site Class and Mapped Short-Period Maximum Considered Earthquake Spectral Acceleration

Site Class	Mapped maximum considered earthquake spectral accelerations at short periods				
	S_S ≤ 0.25	S_S = 0.50	S_S = 0.75	S_S = 1.00	S_S ≥ 1.25
A	0.8	0.8	0.8	0.8	0.8
B	1.0	1.0	1.0	1.0	1.0
C	1.2	1.2	1.1	1.0	1.0
D	1.6	1.4	1.2	1.1	1.0
E	2.5	1.7	1.2	0.9	a
F	a	a	a	a	a

Note: Use straight line interpolation for intermediate values of S_S a: Site-specific geotechnical investigation and dynamic site response analyses shall be performed

Table 3. Values of F_v as a Function of Site Class and Mapped 1 Second Period Maximum Considered Earthquake Spectral Acceleration

Site Class	Mapped maximum considered earthquake spectral accelerations at 1 second periods				
	S_I ≤ 0.1	S_I = 0.2	S_I = 0.3	S_I = 0.4	S_I ≥ 0.5
A	0.8	0.8	0.8	0.8	0.8
B	1.0	1.0	1.0	1.0	1.0
C	1.7	1.6	1.5	1.4	1.3
D	2.4	2.0	1.8	1.6	1.5
E	3.5	3.2	2.8	2.4	a
F	a	a	a	A	a

NOTE: Use straight line interpolation for intermediate values of S_I a: Site-specific geotechnical investigation and dynamic site response analyses shall be performed ig. 1 Fuzzy set for PGA on rock (mean PGA = 0.21g)

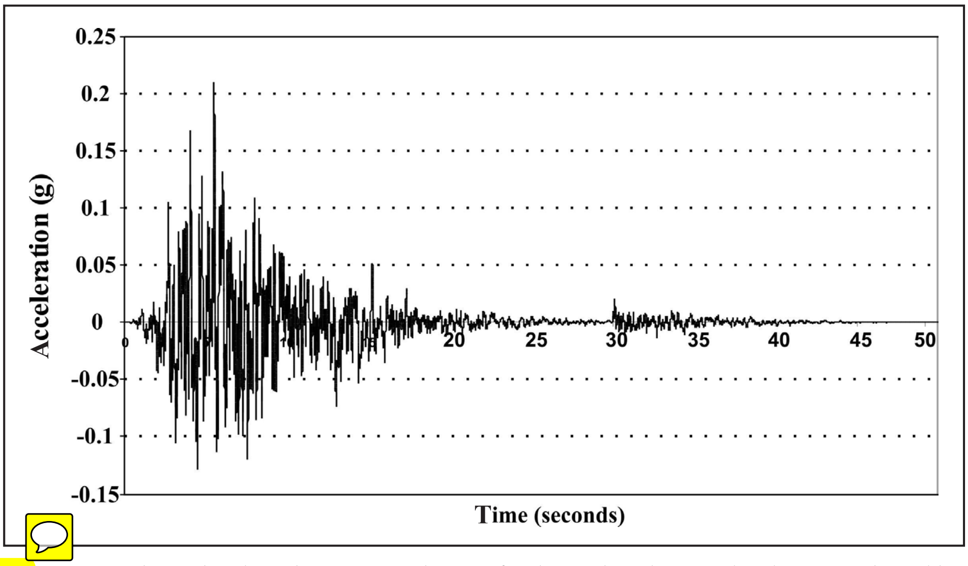

Fig. 2 : Typical simulated acceleration time history for the earthquake considered in example problem

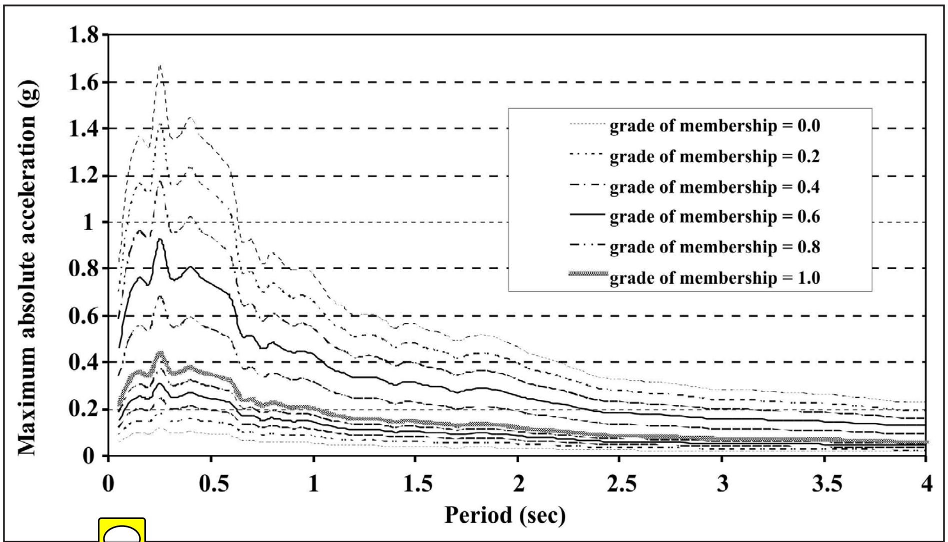

Fig. 3 : Acceleration response spectrum on rock in seismic zone IV (damping = 5)

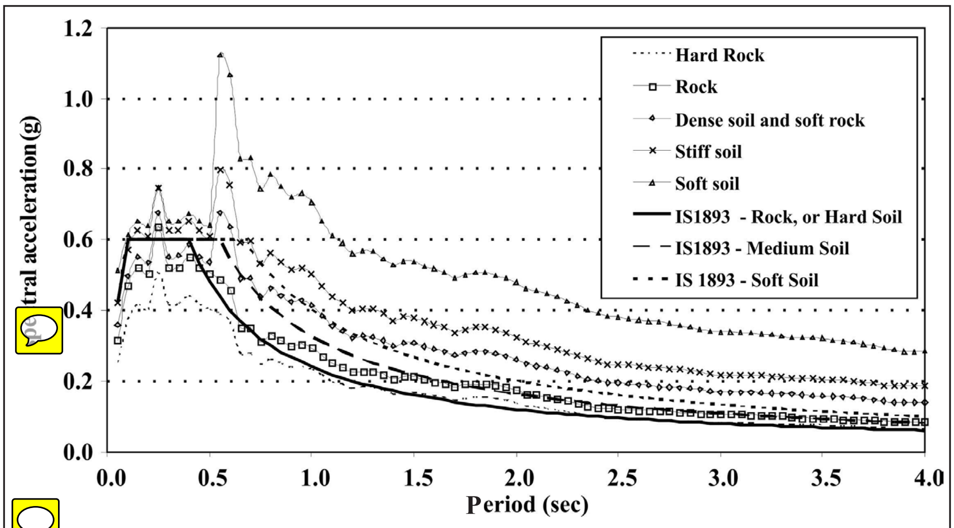

Fig. 4 : Defuzzified acceleration response spectra for different soil profiles in zone IV (damping = 5%)

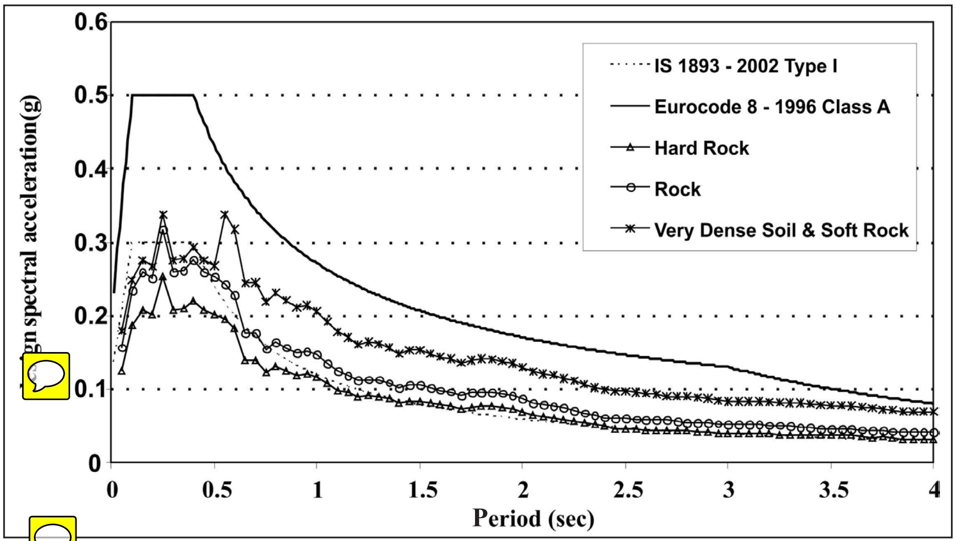

Fig. 5 : Comparison of design spectra for rock and dense soil sites in zone IV (damping = 5%)

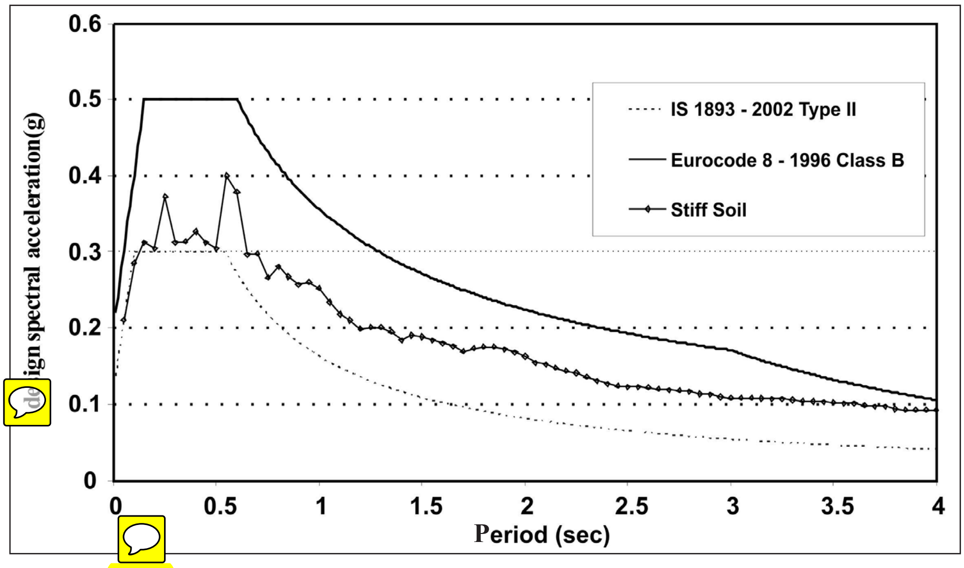

Fig. 6 : Comparison of design spectra for stiff soil sites in zone IV (damping = 5%)

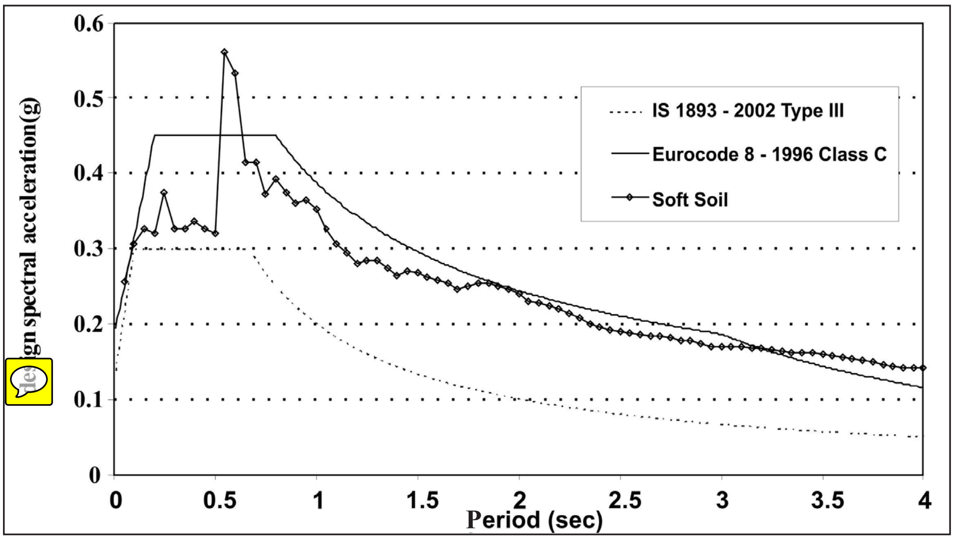

Fig. 7 : Comparison of design spectra for soft soil sites in zone IV (damping = 5%)

the PGA value at the site under consideration, using the attenuation relation given in Newmark and Rosenblueth (1971), has been obtained as 0.21g. It is found from literature that to account for the variations in PGA, it is represented by a lognormal distribution with mean as the value obtained from the attenuation relationship and a high value of COV of up to 0.60 (Campbell and Bozorgnia (1994), Deodatis and Shinozuka (1988)]. This probability distribution is converted into an equivalent triangular fuzzy set (Fig. 1) using the method of least squares (Anoop *et al.*, 2006).

Corresponding to the values at different λ-cut levels of the fuzzy set for PGA, an ensemble of 100 non-stationary accelerograms have been generated using the method proposed by Deodatis and Shinozuka (1988). The earthquake type corresponding to 1940 El-Centro earthquake is chosen for generating the accelerograms. A typical realisation of acceleration time history is shown in Fig. 2. Using these accelerograms, the acceleration response spectra on rock (Site Class B) has been determined (Fig. 3). The response spectrums corresponding to other site classes are also determined by multiplying with the response spectrum on rock with short period and mid-period amplification factors given in IBC 2000 (Tables 2 and 3). The defuzzified acceleration response spectra for the different site classes are shown in Fig. 4. The response spectra given in IS 1893-2002 for different site conditions are also shown in Fig. 4. (Since the mean PGA of 0.21g is near to the zero period acceleration value of 0.24g given in IS 1893 for seismic zone IV, the response spectra corresponding to zone IV is used for the comparison).

The design response spectra for a region in seismic zone IV is also determined using the recommendations given in Eurocode 8. This requires the value of design ground acceleration corresponding to a reference return period of 475 years. The design ground acceleration for seismic zone IV is obtained as 0.20g from the seismic hazard map given by Ravi Kumar and Bhatia (1999). The design response spectra thus developed using the recommendations in Eurocode 8 for different site conditions are given in Figs. 5-7. The design response spectra specified in IS 1893-2002 and that obtained using the proposed methodology are for identical site conditions are also given in these figures. (The response spectra obtained using the proposed methodology are converted into the design response spectra by dividing by a factor of 2, as specified in IS 1893-2002).

3. Discussion of Results

It is noted from Fig. 4 that as the type of soil changes, the period at which the maximum response acceleration occur changes. For soil type A, the peak is around 0.3s, while for soil type C, there are two peaks of almost equal magnitude at 0.3s and 0.65s. For soil type E, while there is a local peak at 0.3s, the maximum peak value occurs at around 0.65s. This is because softer soil sites amplify the spectral acceleration at longer periods. This shows the importance of properly classifying the soil sites.

From Figs. 4-7, it is noted that in the short period regions, the response spectra given in IS 1893 are in agreement with the developed response spectra for similar soil conditions. But, as the period increases, the response spectra given in IS 1893-2002 gives lower values. Also, the individual peaks in the response spectrum developed using the proposed methodology are missing in the response spectrum specified in the codes. This is because the response spectrum in codes of practice is normally developed using a suite of different accelerograms corresponding to different types of earthquakes (ATC, 1996), and averaging the resulting response spectrums. But each individual site has its own characteristic, and will interact with the incoming strong motion in its own way. Averaging the response levels the individual peaks in the response spectrum, which can be unconservative (Seed et *al.*, 2001). It is also noted that the response spectra obtained using Eurocode 8 are more conservative than that obtained using IS 1893. Also, the earthquake time histories that were used for generating the response spectrum given in IS 1893-2002 are not specified. Codes of practice such as ATC 40 (1996) specify the suite of earthquake accelerograms to be considered for developing the response spectra. There is a need to specify such a set of accelerograms for Indian conditions.

4. Summary and Conclusions

While probabilistic approaches have been used internationally in the development of response spectra, the site conditions in the codes of practice are usually defined in a general and qualitative manner using linguistic terms. Thus, both probabilistic and fuzzy uncertainties (arising due to the use of linguistic terms) need to be taken into account while developing the design response spectra. A methodology for generating acceleration response spectrum using fuzzy-stochastic models of earthquake ground motions is proposed in this paper. The proposed methodology takes into account both the random and fuzzy uncertainties in ground motion. The acceleration response spectra for a site in seismic zone IV with different site conditions have been developed using the proposed methodology. The methodology will be useful for developing design response spectrums for the different seismic zones in India for different site conditions.

Acknowledgement

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Probabilistic Analysis of Bi-axially Loaded RC Columns

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Abstract

This paper deals with the investigation on probabilistic analysis of axial load carrying capacity of short rectangular columns subjected to biaxial bending. The effect of variations in compressive strength of concrete and dimensional variations on the axial load carrying capacity of bi-axially loaded short columns are discussed based on the analytical studies carried out. The analytical studies revealed that at low eccentricities of applied load, variation in grade of concrete affects the load carrying capacity of column considerably. At higher eccentricities of applied load about both the principal axes of the column the variation in compressive strength of the concrete or variation in the dimensions of the column will have little influence on the load carrying capacity of the column.

Keywords: Short RC Columns - Bi-axial bending- Monte Carlo Simulation - Squared Coefficient of Variance

1. Introduction:

Traditionally, structural design relies on deterministic analysis. Suitable dimensions, material properties, and loads are assumed, and analysis is then performed to provide a more or less detailed description of the structure. However, due to fluctuations of the loads, variability of the material properties, and uncertainties regarding the analytical model etc. the structure may not perform as intended. To address this problem, Reliability analysis methods which take into account the probabilistic nature of loads, material strengths and dimensions of members have been developed. The methods are rapidly finding application to structural design and reassessment of the safety of existing structures. Although a safety index or reliability index which is a probabilistic measure of safety cannot claim of representing an absolute measure of the failure probability, it does provide an efficient means of comparing the reliability of different structures. The present investigation is aimed at studying the effect of random variations in characteristic compressive strength and dimensions of the column on the load carrying capacity of bi-axially loaded short column. It is known that the load carrying capacity depends on the load eccentricity about both the axes and percentage of longitudinal reinforcement. A brief review of literature relevant with the present study is presented first.

2. Brief Review of Literature:

Mirza and Mac-Gregor (1989) conducted probabilistic study on the strength of reinforced concrete columns and concluded that, the reliability of RC column under the axial load and bending moment is a load path - dependent problem as the column resistance depends on the load eccentricity.

Grant etal.,(1975) reported that the variability of concrete strength is a major contributing factor to the short tied column strength variability in a region of low eccentricity ratios. The variability in the steel strength makes a major contribution to the tied column strength variability when the load eccentricity ratios are high leading to tension failures.

Balaji and Murthy (1999) concluded, as was also reported by Grant etal.,(1975), that the effect of concrete strength is a major contributing factor in the compression failure region of the column and steel strength is a major contributing factor in the tension failure region of column. They also pointed out that the variable cover to reinforcement becomes quite significant and a major contributing factor for the column having smaller depth with higher percentages of steel and eccentricity ratio beyond 0.2.

Three simple criteria that can be used in defining the limit state functions are by the fixing the applied moment, by fixing the applied axial load and by fixing the eccentricity. However, the estimation of the reliability of the RC columns considering the load path should be formulated as a first passage problem that is difficult to solve. A less accurate approach is to treat the problem as a mean up crossing problem.

In most of the studies of the reliability of the RC columns found in the literature (eg.Putcha and Narasimham (1985), Ellingwood (1997),Mirza (1989,1996), Ruiz and Aguilar (1994),Stewart and Attard (1999), Sinha and Kumar (1992), Rajasekaran and Vincent, (1999)), the assumption that the load effects can be treated as random variables rather than stochastic processes is adopted. These studies assume further that the load eccentricity is a deterministic quantity. The fixed eccentricity criterion in fact assumes that the axial load and the bending moment are perfectly correlated. The reliability analysis for such a case is usually carried out in two stages. In the first stage, a probabilistic analysis of the column resistance for the given eccentricity is performed using simulation techniques and distribution fitting methods. In the second stage the reliability analysis is carried out using the probability distribution of the column resistance obtained in the first stage, and one of the well known reliability method such as first order second moment method.

To deal with the correlation between the axial load and the bending moment or the uncertainty in the load eccentricity direct use of the simulation techniques has been considered by Floris and Mazzucchelli (1991) and Frangopol et al (1996).However, the computational effort of the direct simulation method compared with the two stage approach is largely increased. Perhaps this is one of the reasons why the systematic results of reliability analysis that determine to what extent the uncertainty in load eccentricity is important are not commonly reported in the literature.

3. Problem Statement:

The strength of reinforced concrete column may vary due to variations in the material strengths of concrete and steel reinforcement, the cross sectional dimensions of concrete and steel, percentage of steel and cover to reinforcement. The effects of basic variables concrete strength, steel strength and cross sectional dimensions were identified as significant. Present investigation aims at predicting the squared coefficient of variation (Sq. COV) of the load carrying capacity of the bi-axially loaded columns, varying the basic variables concrete strength and cross-sectional dimensions.

3.1 Limit State Equations Considered

The following limit state equations are considered for arriving at the load carrying capacity of bi-axially loaded columns as mentioned in IS: 456-2000.

$$\left(\frac{M_{ux}}{M_{ux1}}\right)^{\alpha_n} + \left[\frac{M_{uy}}{M_{uy1}}\right]^{\alpha_n} \leq 1.0$$

M_{ux1} =Maximum uniaxial moment capacity about x-axis for an axial load of P_u.

M_{uy1} = Maximum uniaxial moment capacity about y-axis for an axial load of P_u.

M_{ux} = Factored moment about x-axis, which is equal to $P_u * e_y$.

M_{uy} = Factored moment about y-axis, which is equal to $P_u * e_x$.

α_n = Related to the ratio P_u/P_{uz}.

P_{uz} = Capacity of the cross section under pure axial load.

$P_{uz} = 0.45\, f_{ck}\, A_c + 0.75\, f_y A_{sc}$

3.2 Methodology:

Monte Carlo simulation has been used to generate random samples of the variables in the present reliability analysis of the R.C columns. Monte Carlo method is found highly suitable to determine the performance of a specified number of synthetic systems and to obtain the overall variability of the system.

To apply Monte Carlo techniques to practical systems it is necessary to,

(a) Develop systematic methods for numerical "sampling" of the basic variable X.

(b) Select an appropriate economical and reliable simulation technique or sampling strategy.

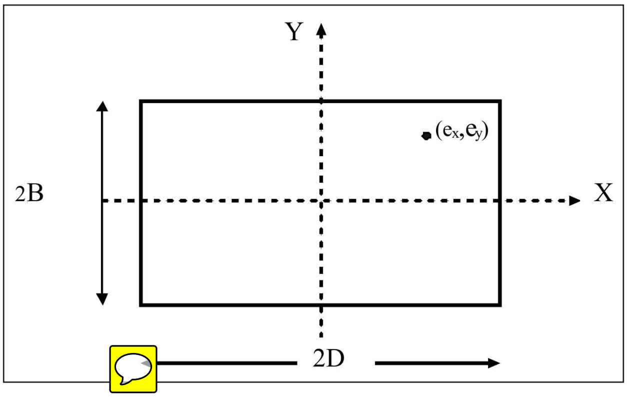

Fig. 1 : Typical Cross section of the column

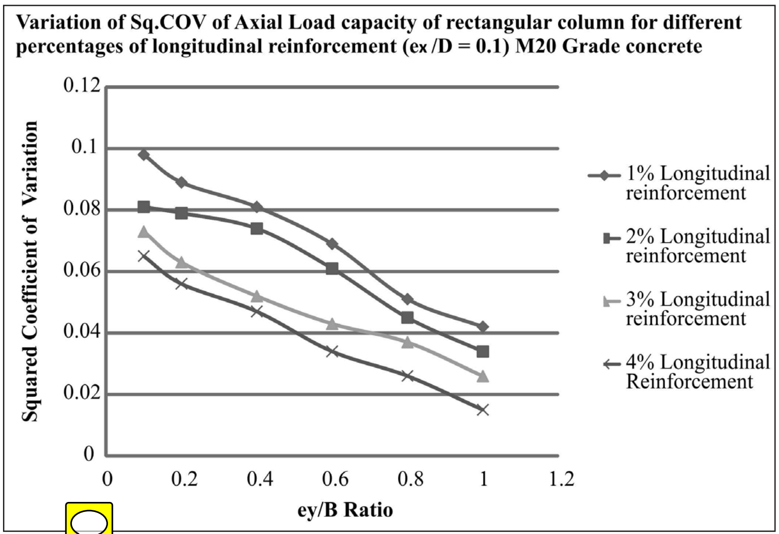

Fig. 2 : Variation of Sq. COV of Axial Load capacity of rectangular column for different percentages of longitudinal reinforcement (e_x/D = 0.1) M20 Grade concrete

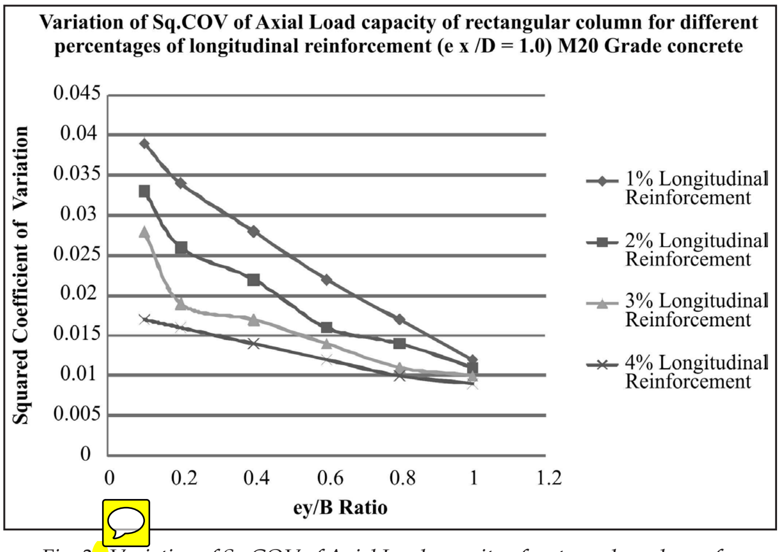

Fig. 3 : Variation of Sq.COV of Axial Load capacity of rectangular column for different percentages of longitudinal reinforcement (e_x/D = 1.0) M20 Grade concrete

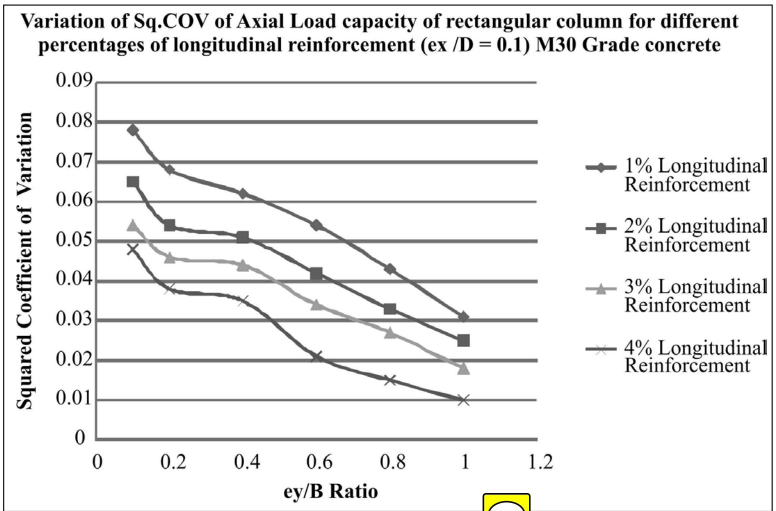

Fig. 4 : Variation of Sq.COV of Axial Load capacity of rectangular column for different percentages of longitudinal reinforcement (e_x/D = 0.1) M30 Grade concrete

(c) Consider the effect of the complexity of calculating system performance indicator G (X) and the number of basic variables on the simulation technique used.

To study the effect of variation of the basic variables the probability distributions of the basic variables considered (Concrete compressive strength, Yield strength of the reinforcement, Percentage of longitudinal reinforcement and dimensions of column size), random values for each of the basic variable have been generated by using a computer programme for Monte Carlo simulation and for every simulation 500 cycles are performed. While studying, the effect of a particular variable, say compressive strength of concrete (f_{ck}), the other basic variables were kept constant.

The standard deviation and mean with respect to strength of concrete and dimensions of the cross-section of the columns were considered from the earlier reported data by Ranganathan(1990). The standard deviation and mean of the Concrete strength, cross sectional dimensions considered in this investigation were presented in Table.1. In this numerical investigation a column of size of 450mm x 230mm was adopted.

A computer program capable of computing bi-axial load-moment interactions diagram with longitudinal steel distributed equally on all four sides of the column has been developed. The concrete section was divided into a number of small strips in both principal directions of size 1mm thickness for evaluating the internal forces and moments.

The forces and moments in concrete and steel at different layers in both directions are obtained by using the stress-strain relationships of concrete and steel as per IS 456-2000. The tensile contribution of concrete is ignored.

By varying the neutral axis depth, the moment – axial load interaction is arrived at. For a given random variable, say compressive strength (f_{ck}), moment carrying capacity varies by varying the

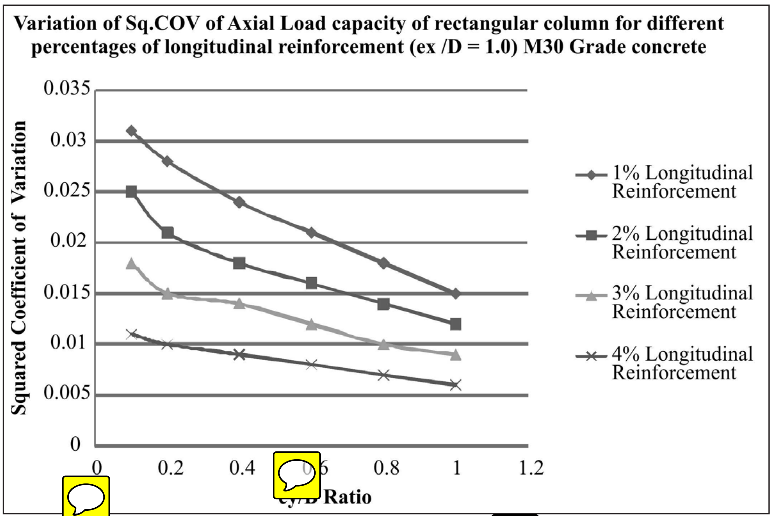

Fig. 5 : Variation of Sq.COV of Axial Load capacity of rectangular column for different percentages of longitudinal reinforcement (e_x/D = 1.0) M30 Grade concrete

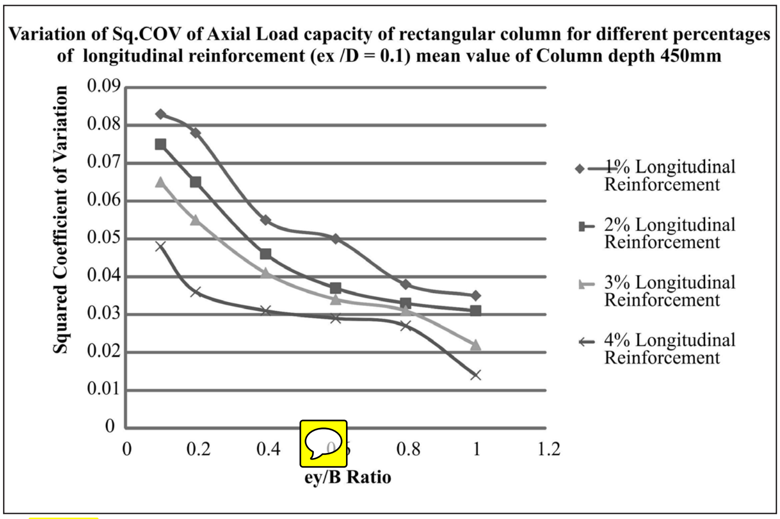

Fig. 6 : Variation of Sq.COV of Axial Load capacity of rectangular column for different percentages of longitudinal reinforcement (mean value of column depth = 450mm) M20 Grade concrete

neutral axis depth. Knowing the moment of resistance of the column under axial load in both principal directions, the load carrying capacity of the column was evaluated considering the eccentricities about the principal axes ($P*e_x$ and $P*e_y$).

For a particular random variable load carrying capacity of the column, vary with respect to the variation of the neutral axis depth. Using Monte Carlo simulation the main random variable characteristic strength of the concrete was varied for about 500 random generations by considering the mean and standard deviation of the corresponding variable from earlier work. For all these random generations the load carrying capacity of the column for given eccentricities in principal directions, the mean, standard deviation, and squared coefficient of variance (Sq.COV), which reflects the level of reliability of axial load carrying capacity of column, were obtained.

The same methodology has been adopted to find the variation in the load carrying capacity of the bi-axially loaded columns for different eccentricities in principal directions.

A column of size 450mm x 230mm has been adopted to perform the reliability analysis; amount of longitudinal reinforcement was varied at 1%, 2%, 3%, and 4%, keeping the grade of steel

Table 1. Mean and Standard Deviation of the Basic Variables Considered in this Investigation.

Sr. No.	Varying Parameter	Nominal Value	Mean	Standard Deviation	Distribution
1.	Grade of Concrete	M20	19.54 MPa	4.56MPa	Normal
2.	Grade of Concrete	M30	31.5 MPa	3.95 MPa	Normal
3.	Depth of Cross section	450 mm	450 mm	7.89 mm	Normal

Table 2. Squared Coefficient of Variation of Axial Load Capacity of Rectangular Column for Different Percentages of Longitudinal Reinforcement (e_x/D = 0.1)

Mean and SD of grade of concrete	Grade of concrete	e_y/B	Sq.COV with various % of longitudinal reinforcement			
			1%	2%	3%	4%
		0.1	0.098	0.081	0.073	0.065
		0.2	0.089	0.079	0.063	0.056
		0.4	0.081	0.074	0.052	0.047
		0.6	0.069	0.061	0.043	0.034
Mean=19.54MPa SD=4.56MPa	M20	0.8	0.051	0.045	0.037	0.026
		1.0	0.042	0.034	0.026	0.015

Table 3. Squared Coefficient of Variation of Axial Load Capacity of Rectangular Column for Different Percentages of Longitudinal Reinforcement (e_x /D = 1.0)

Mean and SD of grade of concrete	Grade of concrete	e_y /B	Sq.COV with various % of longitudinal reinforcement			
			1%	2%	3%	4%
Mean=19.54 MPa SD=4.56MPa	M20	0.1	0.039	0.033	0.028	0.017
		0.2	0.034	0.026	0.019	0.016
		0.4	0.028	0.022	0.017	0.014
		0.6	0.022	0.016	0.014	0.012
		0.8	0.017	0.014	0.011	0.010
		1	0.012	0.011	0.010	0.009

Table 4. Squared Coefficient of Variation of Axial Load Capacity of Rectangular Column for Different Percentages of Longitudinal Reinforcement (e_x /D = 0.1)

Mean and SD of grade of concrete	Grade of concrete	e_y /B	Sq.COV with various % of longitudinal reinforcement			
			1%	2%	3%	4%
Mean=31.5MPa SD=3.95MPa	M30	0.1	0.078	0.065	0.054	0.048
		0.2	0.068	0.054	0.046	0.038
		0.4	0.062	0.051	0.044	0.035
		0.6	0.054	0.042	0.034	0.021
		0.8	0.043	0.033	0.027	0.015
		1	0.031	0.025	0.018	0.010

Table 5. Squared Coefficient of Variation of Axial Load Capacity of Rectangular Column for Different Percentages of Longitudinal Reinforcement (e_x /D = 1.0)

Mean and SD of grade of concrete	Grade of concrete	e_y /B	Sq.COV with various % of longitudinal reinforcement			
			1%	2%	3%	4%
Mean=31.5MPa SD=3.95MPa	M30	0.1	0.031	0.025	0.018	0.011
		0.2	0.028	0.021	0.015	0.010
		0.4	0.024	0.018	0.014	0.009
		0.6	0.021	0.016	0.012	0.008
		0.8	0.018	0.014	0.01	0.007
		1	0.015	0.012	0.009	0.006

Table 6. Squared Coefficient of Variation of Axial Load capacity of rectangular column for different percentages of longitudinal reinforcement (e_x /D = 0.1)

Mean and SD of Column Depth	Grade of concrete	e_y /B and ex/D=0.1	Sq.COV with various % of longitudinal reinforcement			
			1%	2%	3%	4%
Mean=450mm SD=7.89mm	M20	0.1	0.083	0.075	0.065	0.048
		0.2	0.078	0.065	0.055	0.036
		0.4	0.055	0.046	0.041	0.031
		0.6	0.050	0.037	0.034	0.029
		0.8	0.038	0.033	0.031	0.027
		1	0.035	0.031	0.022	0.014

constant (Fe 415). The grades of concretes varied are M20 and M30. The Sq.COV of axial load carrying capacity of column was considered as the representing parameter of the variation. The eccentricities in both principal directions of the column considered were varied from e_x/D=0.1 to 1.0 and e_y/B=0.1 to 1.0

4. Results and Discussion:

4.1 Effect of Percentage of Longitudinal reinforcement:

The Sq. COV of the load carrying capacity of columns with concrete grade M20 for different eccentricity ratios (e_x/D=0.1 and e_y/B= 0.1 to 1.0) for different percentage of longitudinal reinforcement levels (1%, 2%, 3%, 4%) are presented in Table. 2. The variation of Sq. COV of load carrying capacity for the above mentioned parameters is presented in Fig. 2. For these variations, breadth and depth of column cross section and yield strength of the steel is maintained constant. Statistical variation in the compressive strength of concrete is considered. Average compressive strength of concrete is taken as 19.54 MPa and the standard deviation is taken as 4.56 MPa. From the Fig.2 and Table 2, it is clear that, at low eccentricities i.e. e_x/D=0.1and e_y/B= 0.1, the Sq. COV of the load capacity of the column is high, that is around 0.098 for 1% of longitudinal reinforcement, 0.081 for 2% of longitudinal reinforcement, 0.073 for 3% of longitudinal reinforcement, 0.065 for 4% of longitudinal reinforcement.The reason for this can be attributed to the fact that at low eccentricities the column behaves like an axially load column with full cross section area of the column under compression. Thus any variation in the compressive strength of concrete influences the capacity of the section under combined bending and axial load. Thus the increase in variations in the grade of concrete reflects in increasing- the variations in the axial load carrying capacity of the column. The squared coefficient of variance (Sq.COV) of load carrying capacity of the column at higher eccentricities e_x/D=0.1 and e_y/B= 1.0 for all percentages of reinforcement considered decreased from 0.042 (for reinforcement equal to 1% of cross sectional area of column) to 0.015(for reinforcement 4% of cross sectional area of column).

The Sq. COV of the load carrying capacity of columns with concrete grade M20 for different eccentricity ratios (e_x/D=1.0 and e_y/B= 0.1 to 1.0) for different percentage of longitudinal reinforcement levels (1%, 2%, 3%, 4%) are presented in Table. 3 and Fig. 3. From the Fig.3 and Table 3, is clear that, at high eccentricities i.e. e_x/D=1.0 and e_y/B= 1.0, the Sq. COV of the load capacity of the column is very low that is around 0.012 for 1% of longitudinal reinforcement, 0.011 for 2% of longitudinal reinforcement, 0.010 for 3% of longitudinal reinforcement 0.009 for 4% of longitudinal reinforcement. The reason for this decrease in Sq. COV can be attributed to the fact that at high eccentricities the flexural behavior of the column dominates and the concrete in the column cross section which is under tension does not contribute to the capacity of the column. Therefore, any variations in the compressive strength of concrete will have little influence on the variations in the capacity of the section under combined bending and axial load. As expected, the Sq. COV of load capacity of the column is found to decrease with the increase in the percentage of longitudinal reinforcement in all cases considered in this investigation.

From this discussion it may be concluded that variation in the strength of concrete greatly affects the load carrying capacity of the axially loaded columns (or) columns subjected to low level eccentricities about both the axes.It can also be concluded that the increase in the percentage of longitudinal reinforcement decreases the variations in the load carrying capacity of column at higher eccentrics about both axes.

4.2 Effect of the Grade of Concrete:

The variation in the load carrying capacity of the columns with varying percentage longitudinal reinforcement and grade of concrete (M20 and M30) are considered. For M30 grade concrete the average compressive strength and standard deviation are taken as 31.5 MPa and 3.95 MPa, respectively. The variation of the Sq. COV of the load carrying capacity of columns with concrete grade M30 for different eccentricity ratios (e_x/D=0.1 and e_y/B= 0.1 to 1.0) and for different percentage of longitudinal reinforcement levels (1%, 2%, 3%, 4%) are presented in Table. 4 and Fig. 4. From the Fig.4 and Table 4, is clear that, at low eccentricities i.e. e_x/D=0.1 and e_y/B= 0.1, the Sq. COV of the load capacity of the column is high but less compared to the Sq. COV of the load capacity of the column with M20 grade concrete. The decrease in Sq. COV values when M30 grade concrete used instead of M20 grade, is due to the fact that the coefficient of variation for M30 grade concrete is 12.5% while the same for M20 grade of concrete is 23.3%. A lower value of COV for M30 grade concrete may be justified as more care is exercised while adopting higher grades of concrete. From the results presented in Fig.5 and Table 5, it can be concluded that the variation in the load capacity of columns change not significantly when concretes of high strengths with lesser standard deviation are adopted. At higher eccentricities about both principal axes also little variation in axial load capacity of the columns can be observed.

4.3 Effect of Variability in the Cross Sectional Dimensions

The Sq. COV of the load carrying capacity of columns pertaining to this category for different eccentricity ratios (e_x/D and e_y/B) and different percent of reinforcement (1%, 2%, 3%, 4%), for M20 grade of concrete, are presented were in Table 6 and Fig.6. The Mean and SD of the depth are 450 mm and 7.89 mm respectively. From the Table 6 and Fig.6, it is clear that at low eccentricities i.e. e_x/D=0.1 and e_y/B=0.1 the Sq.COV is high that is around 0.083 for 1% of longitudinal reinforcement 0.075 for 2% of longitudinal reinforcement and 0.065 for 3% of longitudinal reinforcement 0.048 for 4% of longitudinal reinforcement. A higher Sq.COV is observed at low levels of eccentricities, the reason for this type of response may be attributed to the fact that at low eccentricities the column behaves like an axially load column with full cross section area under compression effective .Thus variation in the depth dimension has resulted in a higher value of Sq. COV of axial capacity of the column.

4. Conclusions

Based on the analytical investigation carried out on the variations in the load carrying capacity of the biaxial loaded columns the following conclusions have been drawn

1. At low eccentricities of biaxial loaded column variation in compressive strength of concrete affects the axial load carrying capacity of column significantly.
2. At higher eccentricities of load about the both the principal axes of the column the variation in compressive strength of the concrete and variation in the depth of the column will have little effect on the axial load carrying capacity of column.
3. At low eccentricities of axial load about the principle axes, variation in cross-sectional dimensions affects the variation in load carrying capacity of column more.
4. The variation in load carrying capacity of column with M20 grade concrete is more than with M30 grade concrete, when variations in compressive strength of concrete is considered.

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Advances in Interval Finite Element Modelling of Structures

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Abstract

Finite element models of structures using interval numbers to model parameters uncertainty are presented. The difficulties with simple replacement of real numbers by intervals are reviewed. Methods to overcome these difficulties are presented. Recent advances in providing sharp solution bounds for linear static, linear dynamic, and non-linear static analysis of structures are described based on the authors' work. Examples of successful interval finite element solutions to structural problems are given.

Keywords: Finite Elements, Interval Arithmetic, Nonlinear Structural response

1. Introduction

Accounting for uncertainty in modern structural analysis and design is unavoidable. Uncertainties can be classified in two general types: aleatory (stochastic or random) and epistemic (subjective) (Yager et al., 1994; Klir and Filger 1988; Oberkampf et al., 2001). Aleatory or irreducible uncertainty is related to inherent variability and is efficiently modeled using probability theory. When data is scarce or there is lack of information, the probability theory is not useful because the needed probability distributions cannot be accurately constructed. In this case, epistemic uncertainty, which describes subjectivity, ignorance or lack of information, can be used. Epistemic uncertainty is also called reducible uncertainty because it can be reduced with increased state of knowledge or collection of more data. Formal theories to handle epistemic uncertainty have been proposed including Fuzzy sets (Zadeh 1965, 1978), Dempster–Shafer evidence theory (Dempster 1967, Yager et al., 1994; Klir and Filger, 1988), possibility theory (Dubois and Prade, 1988), interval analysis (Moore, 1966), and imprecise probabilities (Walley, 1991).

It is possible to represent uncertainty or imprecision using *imprecise probabilities (*Walley, 1991; Sarin, 1978; Weichselberger, 2000) which extend the traditional probability theory by allowing for intervals or sets of probabilities. In general, imprecise probabilities present computational challenges. By imposing some restrictions, Ferson and Donald (1998) have developed a formal Probability Bounds Analysis (PBA) that facilitates computation; Berleant and collaborators independently developed a similar approach (Berleant and Goodman-Strauss, 1998). Also, related methods were developed earlier for Dempster-Shafer representations of uncertainty (Yager, 1986). PBA can represent uncertainty or imprecision, and it has been shown to be useful in engineering design (Aughenbaugh, J. M., and Paredis, C. J. J., 2006).

Intervals are the basis for analyzing uncertainty using fuzzy sets, possibility theory or imprecise probability theories. In this paper, we will focus on Interval Finite Element Methods (IFEM) itself and not their applications to various theories of uncertainty. In the next section, interval arithmetic will be summarized. The difficulty with naïve replacement of real numbers with interval values will be demonstrated. Approaches to effectively implement IFEM for linear problems are presented in sections 2 and 3. Finally, non-linear IFEM implementations are presented in section 4.

2. Short Review of Interval Arithmetic

Detailed information about interval arithmetic can be found in a series of books and publications such as Hansen (1965); Moore (1966); Alefeld and Herzberger (1983); Neumaier (1990); Rump (1999); and Sun Microsystems (2002). ***In this paper, the notation follows the recommendation of*** (Kearfott et al, 2005). ***Accordingly, interval quantities (interval number, interval vector, interval matrix) are introduced in boldface. Real quantities are introduced in non-bold face.***

2.1 Basic Definitions

A real interval $\mathrm{x} = [\underline{x}, \overline{x}] = \{x \in R: \ \underline{x} \le x \le \overline{x}\}$ is a segment of the real line, where $\underline{x}$ and $\overline{x}$ are the lower and upper bounds of the interval number **x** respectively. The set of real intervals will be denoted by IR. All operations between interval quantities satisfy the fundamental property of *inclusion isotonicity*, That is, if $\circ$ stands for interval addition, subtraction, multiplication, or division, then

$$\mathbf{x}_i \subseteq \mathbf{y}_i \ \ (i = 1,2)\,, \tag{1}$$

implies

$$\mathbf{x}_1 \circ \mathbf{x}_2 \subseteq \mathbf{y}_1 \circ \mathbf{y}_2 \quad \textit{for} \ \circ \in \{+,-,\times,\div\}\,, \tag{2}$$

where $\subseteq$ denotes containment.

Operations with at least one interval operand are, by definition, interval operations, although using the same symbols as real operators. It is easy to see that the set of all possible results $x \circ y$ for $x \in \mathbf{x}$ and $y \in \mathbf{y}$ form a closed interval (for 0 not in a denominator interval), and the end points can be calculated by

$$\mathbf{x} \circ \mathbf{y} = [\min x \circ y, \max x \circ y] \quad \textit{for} \quad \circ \in \{+,-,\times,\div\} \tag{3}$$

It should be noted that the interval arithmetic, does not possess typical algebraic properties, for example, the equality distributive law for real numbers is only an inclusion in the extension to intervals.

2.2 Interval Functions

An interval function is defined as an interval-valued function of one or more interval arguments. Thus an interval function maps the value of one or more interval arguments onto an interval. If we consider a rational real-valued function f of real variables (x_1,..., x_n) and a corresponding interval function **f** of interval variables ($\mathbf{x}_1$,..., $\mathbf{x}_n$), the interval function **f** is said to be an interval extension of f if **f** (x_1,..., x_n) = f (x_1,..., x_n) for any values of the argument variables. That is, if the arguments of **f** are degenerate intervals, then **f** (x_1,..., x_n) is a degenerate interval equal to f (x_1,..., x_n). This definition presupposes the use of exact interval arithmetic when evaluating **f**. In practice, with rounded interval arithmetic, we are only able to compute **F**, viz. an interval enclosure of **f**. Therefore, we have to render

$$f(x_1, \cdots, x_n) \in \mathbf{F}(x_1, \cdots, x_n)\,, \tag{4}$$

even when **f** is an interval extension of f.

An interval function **f** is said to be *inclusion isotonic* if

$$\mathbf{x}_i \subseteq \mathbf{y}_i, \ \ i = 1, \cdots, n \text{ implies } \mathbf{f}(\mathbf{x}_1, \cdots, \mathbf{x}_n) \subseteq \mathbf{f}(\mathbf{y}_1, \cdots, \mathbf{y}_n) \tag{5}$$

This follows from the definition of interval operations (see Eqs. 1-3) that finite interval arithmetic is inclusion isotonic and therefore interval extension of rational functions are inclusion isotonic as well (See Alefeld and Herzberger 1983, Moore et al. 2009, Neumaeir, 1990, Hansen and Walster, 2004).

2.3 Dependency in Interval Arithmetic

Given the fundamental property of *inclusion isotonicity* Eq. (2), the interval-system quality is measured by the width of the interval result, and a narrower (more accurate) result is desirable. However, the width of results may be unnecessarily wide in some occasions due to dependency effect. For example, if the interval function **f** (**x**) = **x-x** is evaluated with **x** = [a, b] = [1, 2], the interval subtraction rule gives the result: **f** (**x**) = [a-b, b-a]=[-1, 1], which contains the exact solution [0, 0], but much wider. The interval arithmetic implicitly made the assumption that all intervals are independent, namely it treats **x-x** as if evaluating **x**-**y**, and **x**, **y** are two independent quantities that happen to have the same bounds. This phenomenon is referred to as the "dependency problem" (Moore 1979; Neumaier 1990; Hansen 1992). Reducing the overestimation is a crucial issue to a successful interval analysis. In general, the sharp results are obtained with the proper understanding of the physical nature of the problem and reduction of the dependency. In the above example, the exact solution could be achieved in evaluating **x-x** as **x** (1-1) = 0.

3. Interval Finite Element

Our intention in this section is to introduce a very brief overview of Interval Finite Element Method (IFEM) and to focus only on the formulation that is used in this work. A detailed review of IFEM can be found in the works of (Muhanna and Mullen 2001, Muhanna, et al 2005, Rama Rao et. al. 2011, Qui and Elishakoff, 1998, De Munck, et. al., 2008, Neumaier and Pownuk, 2007).

A natural idea to implement interval FEM is to apply the interval extension to the deterministic finite element formulation. A straightforward replacement of the system parameters with interval ones without taking care of the dependency problem is known as a naïve application of interval arithmetic in the finite element method (naïve IFEM). Unfortunately, such a naïve use of interval analysis in FEM can yields meaningless and overly wide results (Muhanna and Mullen, 2001; Dessombz et al., 2001). One of the main features of interval arithmetic is its capability of providing guaranteed results. However, it has *the disadvantage of overestimation if variables have multiple occurrences in the same expression as discussed in section 2.3.* Such dependencies lead to meaningless results, and have discouraged some researchers of pursuing further developments of FEM techniques using interval representations.

For example consider the two bay truss shown in Figure 1 with the member cross sectional areas of 0.0001m^2 and with the value of Young's modulus for each element given by the interval [199,201] GPa. Three results are presented in Table 1, a combinatorial (exact) solution, solution computed using the approach devised by the authors (Rama Rao, Mullen and Muhanna, 2011) and a naïve replacement of real numbers by intervals in an existing truss finite element program. As can be seen, the naïve results overestimate the width of interval displacements by 553% to 1745.8% as shown in Table 1. The interval solution without effective consideration of dependency results in meaningless values and reversed signs. However, the authors obtained a very sharp solution with errors in width ranging from 0.714 to 0.821% thus bringing out the effectiveness of their approach.

In efforts to overcome the dependency problem one can recognize two main trends, the first approach can be called the Enclosure Approach (EA) and it provides guaranteed solutions (enclosures to the system response) and it includes a direct solution for the obtained interval linear system of equations after taking care of overestimation reduction based on problem physics (Muhanna and Mullen 1995, Mullen and Muhanna 1999, Muhanna and Mullen 2001, Rama Rao, et al, 2011, Corliss, et al 2007, Pownuk and Neumaier 2007, Popova, et al 2006), and an optimization approach (Koyluoglu et al. 1995, Möller et al 2000, Moens and Vandepitte 2005), while the other approach which can be called an Non-Enclosure Approach (NEA) is based on different forms of sensitivity analysis (Pownuk, 2004) or perturbation methods limited to the first-order terms (Chen, et. al 2002, McWilliam 2000, Qiu, et al. 1996, Qui and Elishakoff 1998) and in the best case to the second-order terms without accounting to the remainders resulting with approximate solutions that do not enclose the actual bounds of the system response and without providing an error estimate.

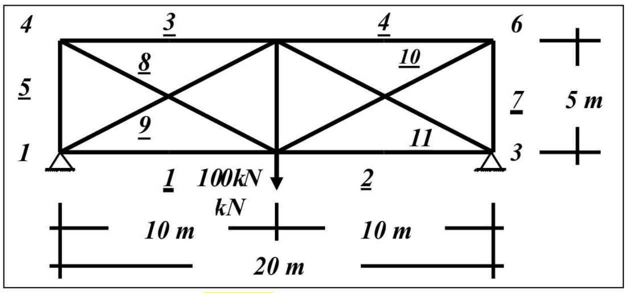

Fig. 1 : Two bay truss.

The most successful approaches for overestimation reduction are those that relate the dependency of interval quantities to the physics of the problem being considered (see, for example, Muhanna and Mullen 1995, 2001 and Zhang 2005). A brief description of the IFEM formulation is presented below, but a detailed explanation of the method can be found in Rama Rao et al. (2011). The two major issues resolved by this formulation are:

1. Reducing of overestimation in the bounds on the system response due to the coupling and transformation in the conventional FEM formulation as well as due to the nature of used interval linear solvers (see Muhanna and Mullen 2001).
2. Obtaining the secondary variables (derived) such as forces, stresses, and strains of the conventional displacement FEM along with the primary variables (displacements) and with the same accuracy of the primary ones.

Table [illegible] Eleven bar truss- selected displacements for 1% uncertainty in the modulus of elasticity (E)

Displacements	U_2(m)×10^4		V_2(m) ×10^1		U_4(m))×10^2	
	U(LB)	U(UB)	V(LB)	V(UB)	U(LB)	U(UB)
Comb. (Exact)	-0.939820	0.939820	-1.00162954	-0.99166308	1.948884	2.00205
Current Solution*	-0.946582	0.946582	-1.00164019	-0.9916026	1.948629	2.0022315
%Error (width)	0.718		0.714		0.821	
Naïve Interval	-5.2	5.2	-1.0835	-0.9095	1.7225	2.2275
%Error (width)	553.3		1745.8		847.9	

The FEM variational formulation for a static discrete structural model is given by minimizing the total potential energy functional

$$\Pi = \frac{1}{2}U^T KU - U^T P \tag{6}$$

which yields

$$\frac{\partial \Pi}{\partial U} KU - P = 0 \tag{7}$$

where Π, K, U, and P are total potential energy, stiffness matrix, displacement vector, and load vector respectively. For structural problems the current formulation includes both direct and indirect approaches. For the direct approach, the strain ε is selected as a secondary variable of interest, where a constraint can be introduced as $C_2\, U = \varepsilon$. For the indirect approach, constraints are introduced on displacements of the form $C_1 U = V$ in such a way that Lagrange multipliers will be equal to the internal forces. C_1 and C_2 are matrices of orders $m \times n$ and $k \times n$, respectively, and m is the number of displacements' constraints, k is the number of strains, and n is the number of displacements' degrees of freedom. We note that V is a constant and ε is a function of U. If we amend the right-hand side of Eq. (6) accordingly will obtain

$$\Pi^* = \frac{1}{2}U^T K_c U - U^T P + \lambda_1^T (C_1 U - V) + \lambda_2^T (C_2 U - \varepsilon), \tag{8}$$

where λ_1 and λ_2 are vectors of Lagrange multipliers with the dimensions m and k, respectively. Invoking the stationarity of Π^*, that is $\delta\Pi^*=0$, we obtain

$$\begin{pmatrix} K & C_1^T & C_2^T & 0 \\ C_1 & 0 & 0 & 0 \\ C_2 & 0 & 0 & -I \\ 0 & 0 & -I & 0 \end{pmatrix} \begin{pmatrix} U \\ \lambda_1 \\ \lambda_2 \\ \varepsilon \end{pmatrix} = \begin{pmatrix} P \\ V \\ 0 \\ 0 \end{pmatrix} \tag{9}$$

The solution of this system will provide the values of dependent variable U and the derived ones λ_1, λ_2, and ε at the same time and with the same accuracy.

The interval formulation used to solve Eq. (9) is an extension of the Element-By-Element (EBE) finite element technique developed by Muhanna and Mullen (2001). The main sources of overestimation in IFEM are the multiple occurrences of the same interval variable (*dependency problem*), the width of interval quantities, the problem size, and the problem complexity, in addition to the nature of the used interval solver of the interval linear system of equations. To eliminate dependencies, the displacements' constraints used in the previous EBE formulation are modified to yield the element forces as Lagrange Multipliers directly and the system strains. Following the procedures given in Rama Rao et al. (2011) we obtain the interval linear system **KU= P**, or explicitly,

$$\begin{pmatrix} \mathbf{K} & C_1^T & C_2^T & 0 \\ C_1 & 0 & 0 & 0 \\ C_2 & 0 & 0 & -I \\ 0 & 0 & -I & 0 \end{pmatrix} \begin{pmatrix} \mathbf{U} \\ \boldsymbol{\lambda}_1 \\ \boldsymbol{\lambda}_2 \\ \boldsymbol{\varepsilon} \end{pmatrix} = \begin{pmatrix} \mathbf{P} \\ 0 \\ 0 \\ 0 \end{pmatrix} \tag{10}$$

Here, **K** is a $(k \times k)$ interval matrix, which contains the individual elements' local stiffness and zeros corresponding to the free nodes' degrees of freedom, where k is the sum of number of elements and free nodes.

The accuracy of the system solution depends mainly on the structure of the matrices in Eq. (10) and on the nature of the used solver. In the current formulation the interval stiffness matrix **K** has the following structure $\mathbf{K} = A\mathbf{D}\,A^T$ where the interval parameters are the elements of the diagonal matrix **D** and columns of matrix A are the eigenvectors of matrix K. The associated solution provides the enclosures of the values of dependent variables which are the interval displacements **U**, interval element forces $\boldsymbol{\lambda}_1$, the multiplier $\boldsymbol{\lambda}_2$, and the elements' interval strains. An iterative interval solver for the system (10) is discussed in the next section. The solver requires that the system be introduce in the following form:

$$(K + B\,\mathbf{D}\,A)\mathbf{u} = a + F\,\mathbf{b} \tag{11}$$

As can be easily seen, Eq. (10) can be reintroduced in the form

$$\left(\begin{pmatrix} 0 & C_1^T & C_2^T & 0 \\ C_1 & 0 & 0 & 0 \\ C_2 & 0 & 0 & -I \\ 0 & 0 & -I & 0 \end{pmatrix} + \begin{pmatrix} A \\ 0 \\ 0 \\ 0 \end{pmatrix} \mathrm{D} \begin{pmatrix} A^T & 0 & 0 & 0 \end{pmatrix} \right) \begin{pmatrix} \mathbf{U} \\ \boldsymbol{\lambda}_1 \\ \boldsymbol{\lambda}_2 \\ \boldsymbol{\varepsilon} \end{pmatrix} = \begin{pmatrix} \mathbf{P} \\ 0 \\ 0 \\ 0 \end{pmatrix} \tag{12}$$

with $\mathbf{K} = A\;\mathbf{D}\;A^T$ which is in agreement with the structure of Eq. (11).

4. Solvers for Interval Linear System of Equations

Any solver for an interval linear system of equations can be used to solve for *unknown vector* in Eq. (10), see (Moore, *et. al.*, 2009, Neumaier 1990, Hansen and Walster, 2004, Rump 1983, 1992, 2001). However, the best known method for obtaining very sharp enclosures of a system with a matrix structure as in Eq. (10), and with large uncertainty, is the iterative method developed by Neumaier and Pownuk (2007). According to this, the interval system in Eq. (10) can be written in the general form:

$$(K + B\mathbf{D}A)\mathbf{u} = a + F\mathbf{b} \tag{13}$$

where $\mathbf{D}$ is diagonal. Furthermore, defining

$$C{:}(K + BD_0A)^{-1} \tag{14}$$

where D_0 is chosen to ensure invertability (often D_0 is selected as the midpoint of $\mathbf{D}$), the solution $\mathbf{u}$ can be written as:

$$\mathbf{u} = (Ca) + (CF)\,\mathbf{b} + (CB)\mathbf{d} \tag{15}$$

To obtain a solution with tight interval enclosure we define two auxiliary interval quantities, viz.

$$v = A\mathbf{u}, \quad \mathbf{d} = (D_0 - \mathbf{D})\mathbf{v} \tag{16}$$

These variables, given an initial estimate for $\mathbf{u}$, are iterated according to

$$\begin{aligned} \mathbf{v}^{k+1} &= \{ACa) + (ACF)\mathbf{b} + (ACB)\mathbf{d}^k\} \cap \mathbf{v}^k, \\ \mathbf{d}^{k+1} &= \{(D_{c0} - \mathbf{D}_c)\mathbf{v}^{k+1} \cap \mathbf{d}^k \end{aligned} \tag{17}$$

until the enclosures converge, and the desired solution $\mathbf{u}$ follows from (15). Observe that not only are the interval displacements $\mathbf{U}$ obtained but also the derived quantities $\boldsymbol{\lambda}_1$, $\boldsymbol{\lambda}_2$, and $\boldsymbol{\varepsilon}$ with the same accuracy.

The EBE procedure for accounting for dependencies among interval values in linear static problems is also effective in structural dynamics using Eigen decomposition and subsequent response spectrum analysis. Algorithms for interval Eigen value extraction are presented in the work of Moens and Vandepitte (2007) and Modares, Mullen, and Muhanna (2006). For symmetric, positive semi-definite interval matrices, the bounds on the Eigen values can be calculated from two real valued analyses, the matrix with all lower values of interval parameters and the matrix with all upper values of interval parameters. Thus, not of the complexity associated with systems of linear equations exist in the structural undamped problem. Interval response spectrum analysis was presented by Modares, Mullen (2013). Interval eigenvectors are calculated using a perturbation procedure (Stewart, G.W. & Sun, Ji-Guang, 1990).

5. Interval Problems with Material Nonlinearity

Methods for the solution of the non-linear system of interval equations associated with non-linear interval finite elements follow existing non-interval equation solution technique. As in the solution of linear systems, concerns about excessive interval widths, require additional considerations in developing equation solving algorithms. Iterations during an interval extension to Newton's method result in large overestimation of interval widths. In the paper by Muhanna, Mullen and Rama Rao (2012), both secant and modified Newton methods are developed for non-linear interval finite elements. In this paper review the interval secant formulation. Given a non-linear constitutive relationship

$$\sigma = f(\varepsilon) \tag{18}$$

An interval extension can be constructed by replacing non-interval variables with intervals and using the basic interval arithmetic operations. (Moore et al. 2009).

$$\sigma = f(\varepsilon) \tag{19}$$

and the expression for the secant modulus, $\mathbf{E}$s becomes

$$\mathbf{E}_S = \frac{\boldsymbol{\sigma}}{\boldsymbol{\varepsilon}} \tag{20}$$

However interval extension might yield an overestimation of the interval secant. In the case of a monotonically increasing function, the interval evaluation of the secant modulus would lead to excessive width. For example, consider the uniaxial stress-strain curve in Figure 2.

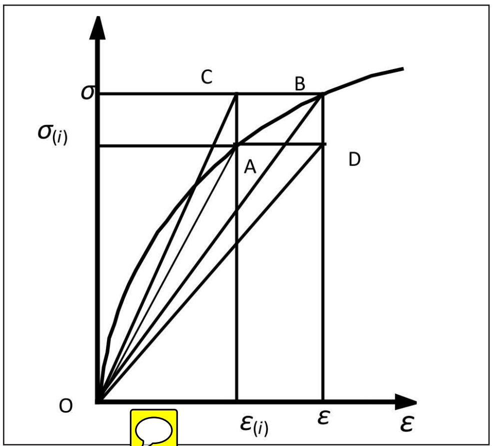

Fig. 2 : Interval secant method.

Table 2. Five bar truss - displacements for 1% uncertainty the load

Method	U_4×10²(m)		V_2×10²(m)		ε_6×10³		ε_9×10³	
	Lower	Upper	Lower	Upper	Lower	Upper	Lower	Upper
Combinatorial	1.87937	2.07786	-10.48393	-9.48226	2.87668	3.18032	-3.55751	-3.21756
Secant	1.87884	2.07835	-10.48393	-9.48209	2.87615	3.18081	-3.55763	-3.21738
Error%(bounds)	0.0282	0.0236	0.000	0.0018	0.0180	0.0154	0.0034	0.0056
Newton	1.87918	2.07797	-10.48397	-9.48180	2.87643	3.18049	-3.55753	-3.21738
Error%(bounds)	0.0100	0.0052	0.0003	0.0048	0.009	0.005	0.001	0.006

Equation (20) would result in lines OC and OD being the bounding secant moduli, but lines OA and OB are the correct bounds based on the physics of the model. Similarly, in the case where the strain interval contains zero, the upper bound of the secant modulus may occur at the zero strain value and not at the limits. Equation (20) must be carefully evaluated by reducing dependency between stress and strain to prevent the division by zero when the strain range contains zero. Constructing algorithms that provide sharp interval values for the secant moduli is critical in obtaining sharp results.

The interval secant solution procedure is straightforward. Starting with an initial Figure 2, Interval secant moduli (from secant moduli calculated from the slope of the constitutive relationship at zero strain, an interval solution is calculated for each element based on the interval load values from Equation (20). The new interval strain values are used to calculate new interval secant moduli for each element and new stiffness matrix is assembled. The solution is repeated and a new strain calculated. The iterations are continued until the L_2 norm of the changes in the upper and lower bounds of the secant moduli is less than a prescribed tolerance. The linear equation solver described in section 3 will be used for the example problems presented. The example problem of two-bay truss is taken up once again (Figure 1) to illustrate the applicability of this approach. The truss is acted upon by a nominal load of 100 kN acting vertically downward at node 2. The non-linear constitutive model is a cubic function with the constitutive model of

$$\sigma = a\varepsilon + b\varepsilon^3 \tag{21}$$

With the constant a and b set to 200 GPa and $10^{13.5}$GPa, respectively. Solutions are obtained using three different approaches viz. modified Newton-Raphson method, the secant method and combinatorial approach.

Table 2 shows the computed values of selected displacements (horizontal displacement U_4 at node 4 and vertical displacement V_2 at node 2) and selected strains (strains ε_6 and ε_9 in elements 6 and 8) using the above approaches. Load interval uncertainty considered is 10 percent (±5% about the mean value of load) and Young's modulus E is deterministic. Overestimation involved in results using the modified Newton-Raphson approach and secant approach is evaluated by comparing the corresponding solutions obtained with the combinatorial approach. Percentage error in the lower and upper bounds of the present solution is computed with reference to the corresponding bounds of the combinatorial solution. It is observed from these tables that error in bounds is quite small for displacements and strains. It is observed that the errors in strains (secondary unknowns) are numerically comparable with the error of displacements (primary unknowns). Thus, the present approach succeeds in obtaining the same level of sharpness for primary and derived quantities.

5. Conclusions

Over the last 15 years, interval finite element has progressed from a computational novelty that predicted correct, but useless bounds on solutions to an elegant tool for providing useful bounds on structural behavior. As in most other areas of engineering, careful use of a computational tool is needed to provide accurate results. In interval finite element methods, the concepts discussed in this paper represent effective use of intervals in structural analysis.

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Safety Assessment of Hybrid Uncertain System: An Overview

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Abstract

This paper reviews the state-of-the-art in safety assessment of system under uncertainty with a special emphasis on the safety evaluation of system characterized by both the probabilistic and the possibilistic uncertain parameters and referred as hybrid uncertain systems (HUS). In doing so, a short glimpse of well-known probabilistic framework of structural safety analysis under random parameters and various possibilistic approaches of reliability analysis are briefed first to pave the way for discussion on the safety evaluation of HUS. To be specific, an attempt has been made to study the various alternatives for safety evaluation of HUS in probabilistic and possibilistic format and tries to scrutinize various options to qualify for safety evaluation of such system compatible with the nature and amount of data feasible to acquire. Finally, concise observations are summarized based on the present review towards safety evaluation for HUS.

Keywords: Safety Assessment, Random Fuzzy set theory, Hybrid uncertain system

1. Introduction

Modern methods of engineering design take into consideration the fact that uncertainty is bound to occur to describe engineering system and it is important to model and adequately treat all available information during analysis and design phases. Typically, the information is originated from different sources: field measurements, experts' judgments, objective and subjective considerations. Over these features, the influences originated from human errors, imperfections in construction techniques, influence of boundary and environmental conditions are added. All these aspects can be brought back to one common denominator: *presence of uncertainty***.** The uncertainty can be viewed as a part or class of imperfection in the information that attempts to model system behaviour in the real world. Different types of uncertainty and imprecision including physical randomness of data, choice of a model, numerical accuracy of calculations or lack of clarity in the objectives and their consequences in the process of modelling and analysis are discussed in French (1986). In real application it is classified in such a way that a mathematically founded and realistic description is ensured in the structural analysis and safety evaluation (Möller *et al.* 1999). For modelling purpose, the uncertainty is usually viewed in two categories, namely aleatory and epistemic. The aleatory uncertainty is classified as objective and irreducible uncertainty with sufficient information on the input uncertain data. These are inherently connected to the problem at hand and cannot be influenced by the observer. The epistemic uncertainty is a subjective and reducible uncertainty that stems from the lack of knowledge about the input uncertain data. It arises from the cognitive sources involving the definition of certain parameters, human errors, inaccuracies, manufacturing and measurement tolerances etc. In simple words, the objective uncertainty can be considered as the tendency of an event to occur whereas the subjective uncertainty is concerned with the ability to occur.

The profession has accepted the fact that the existence of uncertainty cannot be avoided in the analysis and design of engineering system. It is now well recognized that considering the existence of uncertainty in the analysis and design leads to a more cost effective solution rather than when it is planned to eliminate or greatly reduce them for a desired design that will be safe, reliable and robust against uncertainty. Thus, the consideration of uncertainty in engineering analysis and design is gaining increasing importance in the profession. The uncertainty quantification in a typical engineering decision making process involves: characterization of uncertainty of various system parameters including external environment; propagation of this uncertainty through engineering models and computational tools. The first step of characterization of uncertainty involves development of methodologies to model uncertainty of both the epistemic and the aleatoric type. Regardless of the type being considered, the characterization process depends on experimental research and expert judgement. The outcome of the process is in the form of probability distribution function (*pdf*), membership function (*mf*), interval bounds etc. depending on the quantity and quality of data. The propagation of uncertainty mainly involves two aspect i.e. the response analysis of system considering the uncertain input parameters and the associated safety analysis compatible with the decision making process. The response analysis under uncertainty is generally performed through numerical model and various methods like the stochastic finite difference method, stochastic finite element method, stochastic boundary element method, fuzzy finite element method etc. The response surface method (*RSM*) based various metamodel strategies are also quite common to approximate expensive computer simulations (Box and Draper 1987, Jin et al. 2001). More details on response analysis of uncertain system may be seen elsewhere (Schuëller 1997, 2000). The safety of structure is assessed through reliability analysis either in probabilistic or in possibilistic format depending on the nature of available information. The present paper focuses on the various methods of safety evaluation of structural system under uncertainty with a special emphasis on the safety evaluation of structure when it is characterized by mixed uncertain parameter (both probabilistic and possibilistic) and referred as *hybrid uncertain systems(HUS)*. In doing so, a short glimpse of well-known probabilistic framework of structural safety analysis under random parameters and various possibilistic approaches of reliability analysis are briefed first to pave the way for discussion on safety evaluation of *HUS.* To be specific, an attempt has been made here to study the various alternatives for safety evaluation of *HUS* either in probabilistic with or in possibilistic format and tries to scrutinize various options to qualify for safety evaluation of such system compatible with the nature and amount of data feasible to acquire. Finally, concise observations are summarized based on the present review towards safety evaluation for *HUS*.

2. Safety Analyses of Structures under Uncertainty

In last twenty years or so, many new methods have been developed to deal with the imperfection in input data. The large number of models reflects that there exist many aspects of imperfection and the probability theory is not the unique normative model that can be coped with all of them. In fact, numerous methods are used to deal with the uncertainty in natural sciences and engineering. These include from the probability theory and its variants (Bayesian theory, reliability theory), to multi-valued logic, fuzzy and related possibility theory, interval analysis etc.

2.1 The Probabilistic Approach

The development of the theory of structural reliability has a long history. For a long time probability theory was the only theory used to quantify uncertainty and reliability analysis. Even now, probabilistic methods are almost exclusively used in industry. The development in the field is quite extensive. In fact there are numerous well known books on this e.g. Thoft-Christensen and Baker 1982, Madsen et al. 1986, Tichy 1992, Ditlevsen and Madsen 1996, Melchers 1999, Haldar and Mahadevan 2000a,b, Choi et al. 2007, Lemaire 2009). In probabilistic approach, the assurance of performance is referred to as the reliability. The first step for evaluation of reliability of a system is to decide on specific performance criteria and the relevant load and resistance parameters and the functional relationship among those define respective performance criterion. In general, for n number of random variables represented by a vector **X**, the performance function is defined as: Z= g(**X**). The failure surface or the limit state of interest can then be defined as Z=0. This is the boundary between the safe and unsafe regions representing a state beyond which a structure can no longer fulfil the function for which it was designed. The probability of failure (p_f) is given by the multidimensional integral,

$$p_f(Z<0)=\int .. \int_{g(X)<0} f_X(X)dX \tag{1}$$

Where, $f_X(X)$ is the n-dimensional joint *pdf* of the basic random variables. The full distribution approach of computation of probability of failure (p_f) by above equation requiring the joint *pdf* of random variables is almost impossible; moreover, evaluating the multiple integral is a formidable task. The Monte Carlo Simulation (*MCS*) technique is historically used as the robust alternative to compute p_f with known *pdf* of the random variables (Marek et al. 1996, Ferson 1996). But it requires a number of deterministic analyses ranging between few hundreds to tens of thousands depending on the magnitude of p_f. Normally, the second moment based approximation methods i.e. the first order reliability methods (*FORM*) and the second order reliability methods (*SORM*) are applied to obtain a reasonable estimate of p_f with significantly lower computational effort. In the *FORM* and *SORM*, typically a most probable point (*MPP*) of failure is found in a standard normal space, **U**. Any set of continuous basic random variables, **X**, is transformed to **U**-space using a one to one transformation i.e. **U**=T(**X**). The *MPP*, $\mathbf{u}^*$, lies on the limit state surface g(**X**) = G (**U**) = 0, and it is the closest point on the limit state surface to the origin in **U**-space. The *MPP*, $\mathbf{u}^*$, can be obtained by solving the optimization problem,

$$\min \quad \beta = \sqrt{\mathbf{U}^T\mathbf{U}} \quad \text{such that} \quad G(\mathbf{U}) = 0 \qquad (2)$$

Various algorithms are available to solve the problem i.e. the Hasofer-Lind, the Rackwitz-Fiessler algorithm using Newton-Raphson root solving approach, the sequential quadratic programming etc. Once the *MPP* is obtained, the failure probability can be estimated by using the *FORM* or *SORM* algorithm. *FORM* is based on a linear approximation of the limit state built at the *MPP* in **U**-space and p_f is obtained as $\Phi(-\beta)$, where Φ is the standard Gaussian cumulative distribution function (*CDF*). The reliability index obtained by using *FORM* can be exactly related to p_f if all the variables are statistically independent and normally distributed. For any other situation corrections are required involving information on the distributions of the random variables (Rackwitz and Fiessler 1978). The *SORM*, a product of the first-order result and a curvature correction (Breitung 1984, Kiureghian et al. 1987) is based on the fact that a second-order expansion of the limit state surface is better than a first-order expansion i.e. a natural extension of FORM. In this regard, an excellent summary and review on various theory and methods of structural reliability and their applicability may be seen in Rackwitz (2001).

2.2 The Possibilistic Approach

The probabilistic approach of safety analysis of structural system as discussed in the previous section considers some of the variables as random and rests are assumed to be deterministic. Such approach attains its limitation when insufficient reliable data are only available to describe the real life systems with the aid of *pdf.* Moreover, the real world problems are more complex than their corresponding mathematical model and to compensate this gap, some linguistic explanation occasionally adds to the results obtained through the models. The effectiveness of probabilistic approach is thus lost to deal with such non-probabilistic uncertainty. In order to quantify such information, it is desirable to apply the uncertainty measure on the basis of existing available data with the additional information from expert knowledge and experience. This has led to the development of various possibilistic methods. The related developments are briefly summarized in the following.

2.2.1 The Possibility/Fuzzy Set Theory Based Approaches

The possibility is an alternate approach to the probability, initially introduced to model the uncertainties when the available information is linguistic. Zadeh (1965) introduced the notion of fuzzy sets, based on the idea of degree of *mf* to an imprecisely defined set and used it as a basis for possibility (Zadeh 1978). It is based on the possibility distribution defined by *mf* obtained from numerical data along with expert knowledge and experience. For a fuzzy set $\tilde{x}$, the *mf* is defined as $\mu_{\tilde{x}}(x)$ for all *x* that belongs to the domain **X** i.e. $\tilde{x} = \{(x, \mu_{\tilde{x}}(x)) | x \in \mathbf{X}, \mu_{\tilde{x}}(x) \in (0,1)\}$. For a fuzzy variable *F*, a *mf* is typically described in Fig.1a. The core comprises those element *x* of the universe such that $\mu_{\tilde{x}}(x) = 1$, the support comprises those elements **x** of the universe such that $1 > \mu_{\tilde{x}}(x) > 0$. A convenient way to represent the fuzzy variables is the α-discretization method. The α-cut subsets of *F* are defined as: $F_\alpha = \{x \in \mathbf{X}, \mu_F(x) \geq \alpha\}$.

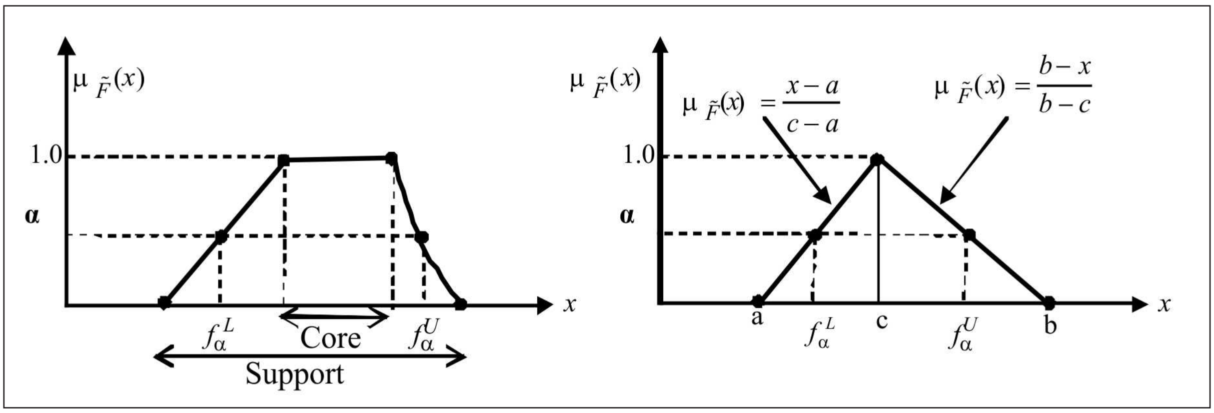

Fig. 1 : (a) The fuzzy distribution and a-cut description (b) A triangular fuzzy mf

$\{f : f \in F_\alpha\}$

, $0 \le \alpha \le 1$. For i[th] α-cut level, the upper and lower bounds f_α^L and f_α^U as depicted in Fig. 1a are given by: $f_\alpha^L = \min\{f : f \in F_\alpha\}$, $f_\alpha^U = \max\{f : f \in F_\alpha\}$. For a given *mf* of a fuzzy variable F as shown in Fig.1(b), one can obtain the two interval values at any specified α-cut level, , $F_\alpha = (f_\alpha^L, f_\alpha^U) = [\alpha c + (1-\alpha)a, \alpha c + (1-\alpha)b]$.The uncertain input variables (x_i) described by the fuzzy *mf*, $\mu_{\tilde{x}_i}(x)$ is transformed to fuzzy output response through α-discretization. Normally, the vertex method is used. The optimum output values z_j are described by the *mf*, $\mu_{\tilde{z}_j}(z_j)$. The uncertain failure functions $\pi(z_j)$ is considered to be the possibility distribution function and describe the possibility of failure with respect to j-th limit-sate condition which may be directly defined or determined from the *mfs*. The failure possibility for j-th limit state is obtained by evaluating the *mf*, $\mu(z_j)$ of the output fuzzy results and the uncertain failure condition $\pi(z_j)$ with the aid of

$$\pi_f = \sup_{z_j \le Z_j} \min(\mu(z_j), \pi(z_j)) \tag{3}$$

Possibility was also defined as a special case of the plausibility measure, which is used in the evidence theory. According to this definition, if the body of evidence about a set of events is nested, then the plausibility of each event reduces to possibility. On the other hand, if the body of the evidence consists of singletons, then plausibility reduces to probability. Based on this interpretation of possibility, Dubious and Prade (1997) has shown that a normalized possibility distribution π(ω) can be effectively represented by a fuzzy number from its *mf* and the fuzzy number can be used to select a class of probability measure for which possibility and necessity represent the upper and lower bounds of the probability of each event. The concept of credibility measure (Liu and Liu 2002) to define a self-dual measure is worth mentioning in this regard. Credibility theory, founded by Liu (2004) and refined by Liu (2007), is a branch of mathematics for studying the behavior of fuzzy phenomena. The foundation and development of fuzzy sets theory, fuzzy modeling and computational methods and its application are well documented in (Klir and Yuan 1995, Dubois et al. 1997).

Brwon (1979) first applied the fuzzy measure concept with classical structural reliability theory to obtain more reliable failure mode. Subsequently, Shiraishi and Furuta (1983), Yao and Furuta (1986) used fuzzy logic in structural reliability application. The outline of fuzzy theory demonstrating its applicability in various structural engineering applications is presented in Furuta (1995). Cremona and Gao (1997) presented an alternative to the probabilistic theory using a new confidence measure: the measure of possibility to estimate the distribution of possibility of failure. Möller *et al*. (1999) developed a safety assessment method by transferring the fuzzy input variables described by the *mf* to fuzzy output response through α-cut using efficient optimum vortex method. Anoop et al. (2005) demonstrated the use of fuzzy system reliability analysis method for reliability-based optimal design of structural systems. Starting from the fundamentals of fuzzy mathematics to fuzzy structural analysis and subsequently fuzzy reliability analysis is well covered in the text on fuzzy randomness (Möller and Beer 2005). Further on fuzzy structural analysis may be may be seen in some of the recent publications (Hanss et al. 2010, Sadeghi, et al. 2010, Reuter and Schirwitz 2011).

2.3 Interval Analysis and Convex Modelling

In many cases, for example in preliminary design phases, even though some experimental data are available, it is not enough for reliable construction of *mfs* or *pdfs* to characterize the uncertain parameters. The available data can be used, particularly in combination with the engineering experience, to set some tolerances or bounds only. A typical example is the uncertainty in the parameters arising from the manufacturing tolerances, materials defects and variation in the operating conditions or errors in the observations. In such cases, the uncertain parameters can be well modelled using the non-probabilistic convex models of uncertainty (e.g. intervals, ellipse or any convex sets) termed as uncertain but bounded (*UBB*) model. BenHaim and Elishakoff (1990), BenHaim (1995) proposed a version of worst-case design based on convex models for design problems where there is scarce information about the uncertain variables. The subsequent developments of convex models and interval analysis methods of uncertainty for structural reliability analysis in which the bounds on the magnitude of uncertainty is only required are extensive (Mullen and Muhanna 1999, Ganzerli and Pantelides 2000, Qiu 2003, Adduri, and Penmetsa 2007). Recent studies with regard to interval *MC* methods for structural reliability (Zhang et al. 2010), correlation analysis of non-probabilistic convex model and its corresponding structural reliability technique (Jiang et al. 2011), interval importance sampling method for finite element based structural reliability assessment under parameter uncertainties (Zhang

2012) are notable.

The interval method of analysis seems to be a logical alternative when the parameters required to create the probabilistic models cannot be precisely determined due to lack of data. Interval analysis usually considers rectangular model which encloses all the possible combinations of the uncertain variables i.e. consists of all the possible probabilities that are consistent with the available information. It is basically a worst scenario method since all the *UBB* variables vary independently and thus may reach their extreme values simultaneously lead to overly conservative design. The ellipsoid model considers all the variables to be correlated with each other, which excludes the extreme combination of the uncertain parameters and thus avoids over-conservative designs. However, in reality, only part of these *UBB* variables is actually correlated while some others vary independently. Therefore, a more realistic option is to divide all the *UBB* quantities into groups and treat them with a multi-ellipsoid convex model (Luo *et al*. 2008).

2.4 Probabilistic or Possibilistic Safety Analysis

The advantages of probability-based safety analysis are now well known and have been accepted broadly by the profession (Sexsmith 1999). In fact, it has become an essential aspect of structural safety analysis. Various second moment based approximations (*FORM/SORM*) approaches, well matured and established in the profession can provide good estimate of p_f with significantly lower computational effect. However, the second moment methods may yield erroneous estimate of the failure probability for problem having multiple *MPP* (e.g. as in case of structural dynamic problems). The errors using FORM/SORM in view of the large uncertainty in selecting the appropriate stochastic model and its parameters, various RSM based *MCS* methods are becoming more attracting in the light of enormous development in the field of computational science. In this regard, the most significant criticism on the widespread use of classical reliability methods is that the information input in the analysis has to be in a precise probabilistic format and the limit state function through which this information is propagated is a precise model. The concept of predictive reliability index that provides a measure of uncertainty in estimated reliability index due to parameter uncertainties are worth mentioning (Kiureghian 2008). It has been shown (Ben-Haim and Elishakoff, 1990) that even small errors in the statistical parameters may have large effects on the computed p_f, especially when these probabilities are very small. The statistical distributions of the parameters, good information on correlations etc. are seldom known for all random variables in real life design problems. Scarcity of data available necessitates strong assumptions which may be sensitive enough in terms of safety of the system. Thus possibilistic methods requiring less information yet can provide a measure of reliability is becoming attractive particularly, at early stage of design. The subjective uncertainty representing the design imprecision and inexactness in choosing among design alternatives usually dominates the preliminary design configuration. Moreover, the probabilistic approach is difficult to make more conservative to protect the design to inaccuracies made due to lack of information. But, it is easy to increase the degree of conservatism in possibilistic approach [Sophie 2000]. However, the issue of switching from probabilistic to possibilistic approach by justifying whether the information is little is not very clear. It is also important to note that the possibility theories are of little use in design of system with a large number of failure modes that are known to be independent (Chen et al 1998). As the number of modes increase the size of failure region increases lead to larger probability of failure, but the possibility of failure remains the same as it is the possibility of a single mode (the maximum one). Possibility of failure effectively imposes a factor of safety (>1) on the probability of failure in general. This conservatism would certainly ensure performance but could adversely affect the optimum cost. With the progress of iterative design process, these types of uncertainty reduced gradually, but the objective uncertainty remains throughout the design process. Though, the valuable comparisons between these two methods are available in the literature (Sophie 2000, Nikolaidis *et al.* 2004), no work seems to be available to find the relationship between the two approaches. Moreover, as such there is no consensus about what method should be used in many real life problems. The general understanding is that if the uncertainties are modelled accurately, the probabilistic methods are better than its counterpart for efficient design (Nikolaidis *et al.* 2004). But, it has been repeatedly invoked in literature that possibility is a better choice in cases of scarcity of data as it is not only safe but also simpler to apply.

3. Safety Assessment under Hybrid Uncertainty

The probabilistic and possibilistic approach of safety analysis of structures as discussed in the previous sections have been developed independently i.e. the former is to consider the aleatoric (random) uncertainty and the latter is for epistemic (non-stochastic) uncertainty. Interestingly, most of the observations on both the approaches are based on the fundamental assumption that the system input information are either all probabilistic or all possibilistic in nature. But in many real situations, some of the input parameters to model the mechanical system are possibilistic in nature while for other parameters the information is sufficient to model those as random. Usually probabilistic or possibilistic approach of safety analysis considering all the variables as single type (either probabilistic or possibilistic) involves gross assumptions and poses serious restriction on the necessary flexibility to the designer, particularly at early design stage. However, for modelling of structure, it is desirable for a designer to describe some of the uncertain parameters as probabilistic (random) and few others as possibilistic (fuzzy, UBB type) depending on the nature of available data so that less assumptions are involved. This requires safety evaluation of *HUS*. There are quite a number of concepts such as p-boxes, coherent lower and upper bounds on probabilities, random set approaches, fuzzy probabilities etc. to deal with mixed probabilistic and non-probabilistic information for reliability assessment of *HUS*. The related developments that have been taken place are discussed here under the subheads of: random fuzzy analysis, transformation approaches for reliability analysis of *HUS*, and reliability analysis of system characterized by random and UBB parameters. The literatures of safety analysis in the framework of Bayesian approach and imprecision theory (Augustin and Hable 2010) are not included in the present review. There is a class of literatures on optimization under hybrid uncertainty (Du and Choi 2008), which has also been excluded from the present review. These aspects perhaps need separate considerations.

3.1 Random Fuzzy Analysis

The early attempt to deal with probabilistic and possibilistic parameters in reliability analysis of mixed uncertain system was by Cai et al. (1995) where random variables are taken as fuzzy type whose *mf* are random variables. Utkin et al. (1996) presented a procedure for reliability analysis of systems whose components can be described in both probabilistic and possibilistic context i.e. fuzzy variables are taken as random variables whose distribution functions are taken as fuzzy type. Yubin et al. (1997) analysed the fuzziness between structural reliable state and failure state. They propose an intermediary transition between reliable and failure state i.e. there should be a fuzzy interval with transition boundaries and introduce the fuzzy limit state function to evaluate the structural fuzzy random reliability for a given reliable level. Rao et al. (1998) introduced the concept of hybrid uncertain mean and variance for unified solution in finite element method (*FEM)* to tackle hybrid uncertainties in which the system parameters are modelled as random and fuzzy type simultaneously. However, the new measures are finally found to be more like fuzzy information than stochastic and found some use at early stage of design. Langley (2000) has shown that the same numerical algorithm could be used for finding both the probability and the possibility of failure, modifying only the function to be minimized. Bing et al. (2000) proposed a method for fuzzy reliability analysis where the fuzzy linear regression model is used in conjunction with *FEM*. The *FEM* code is used to obtain a series of stress values, and then the fuzzy regression function of the stress is obtained through fuzzy linear regression analysis. Similar to the stress-strength inference model in the classical reliability theory, the fuzzy stress-random strength is proposed to evaluate the fuzzy random reliability of mechanical structure. Mőller et al. (2003) introduces the theory of fuzzy random variables to describe uncertainty with the characteristic fuzzy randomness. The ordinary random variables are contained in the fuzzy random variables as a special case uniquely defined by the mean values of the fuzzy realization $\tilde{X}$ (*mf* =1.0). By means of α-discretization, the fuzzy *pdf* determination reduces to computation in ordinary probability space. The limit state surface is specified by the computational model results in fuzzy limit state surface $\tilde{g}(x) = 0$ in the original space of the basic variables. For fuzzy FORM analysis, the fuzzy random variables $\tilde{X}_i$ having fuzzy distribution function $\tilde{F}(x)$'s are transformed to standard normalized variables $\tilde{Y}_i$. An evaluation of the fuzzy limit state surface $\tilde{h}(y) = 0$ yields the fuzzy design point $\tilde{y}_b$ and the fuzzy reliability index $\tilde{\beta}$ defined by the associated *mf* obtained by α-discretization. Jiang and Chen (2003) developed a computational model to obtain the fuzzy reliability of mechanical component by considering the general stress as a random variable and the general strength as a fuzzy variable. The generalized strength is viewed not only as fuzzy but also has randomness. The probability that the general stress is larger than the general strength at threshold λ is obtained by transferring the fuzzy variable to random variable. The basis of the method is that the fuzzy reliability can be obtained with the conventional probability by use of mathematical transition. Chakraborty (2003) has reviewed the various safety assessment alternatives under uncertainty. Moens and Vandepitte (2005) critically reviewed the emerging non-probabilistic i.e. the interval and fuzzy approaches for uncertainty treatment in *FEM* and opined that the non-probabilistic concepts are complementary rather than competitive to the classical probabilistic approach. Karimi and Hullermeier (2005, 2007) presented a procedure for assessing the risk of natural disasters under highly uncertain conditions where neither the statistical data nor the physical knowledge required for a purely probabilistic risk analysis are sufficient. The theoretical foundation of the study is based on employing fuzzy set theory to complement the probability theory with an additional dimension of uncertainty.

More recently Random Set Theory (*RST*) based methods have been proposed to consider mathematical model of uncertainty when the information about uncertain parameters of a system is not complete or when the result of each observation is not point-valued but set-valued, so that it is not possible to assume the existence of a unique probability measure (Tonon and Bernardini 1998, Tonon et al. 2000a, oize 2005, Adhikari 2007, 2008). It has been used to perform reliability analysis when data are affected by both imprecision and randomness (Tonon et al. 200b). The *RST* allows one to obtain the upper and lower bounds on the probability of occurrence of an outcome. The approach has been found to be a general framework for computation under uncertainty because interval analysis, convex models, fuzzy measures, and probability measures (when the information becomes more and more precise) can be viewed as a particular case of evidence theory and *RST* approach to uncertainty.

3.2 Transformation Approach for Safety Analysis of HUS

In reliability analysis of *HUS*, the limit state function of the related reliability analysis problem involves both the probabilistic parameters described by the associated *pdf* and the possibilistic variables usually described by the associated *mf* of the fuzzy variable or bounds of the UBB type variables. It is generally realized that either the probabilistic or the possibilistic approach is not compatible for reliability analysis of structures under such situation. To make the analysis compatible with the reliability analysis in the probabilistic format or in the possibilistic format, one needs to express the performance function either in terms of the random variables or in terms of the fuzzy variables depending on the approach of analysis desired to apply. Various transformation methods have been emerged in the literature to transform the possibilistic variables to equivalent probabilistic variables or vice versa and seem to be potential for reliability analysis of *HUS*.

Following the interpretation of possibility distribution based on the evidence theory, Dubious and Prade (1991) defined an equivalent class of *pdf* where the lower and upper bounds of the probability are shown to be the possibility and the necessity. Based on this, Ferrari and Savoia (1998), Savoia (2002) describe compatible *CDF* in which the left boundary coincides with the increasing branch of the *mf* of the fuzzy variables and the right side boundary coincides with its complement i.e. the decreasing branch. The upper and lower bound *CDFs* corresponding to a fuzzy distribution are conceptually clarified in figure 2. The upper and lower bound *pdfs* can be obtained by differentiating the associated *CDF*. The *pdfs* compatible with the fuzzy description lies in between these two distributions and infinite number of distributions may exist. The bounds on failure probability obtained by the approach are found to be too far apart and too conservative having little use for practical safety analysis (Chakraborty and Sam 2007). This is obvious as the evidence theory gives conservative estimate of the upper and lower bound of the *CDF*. However, these can be used to serve as a check whether the reliability analysis results obtained from various transformations based algorithms following the consistency principle are within such evidence theory based conservative estimate of reliability bounds.

Using the basic concept of entropy, the fuzzy imprecision can be transformed to random uncertainty or vice versa (Dubois et al. 2004). The basis of this transformation is that the measurement is invariant

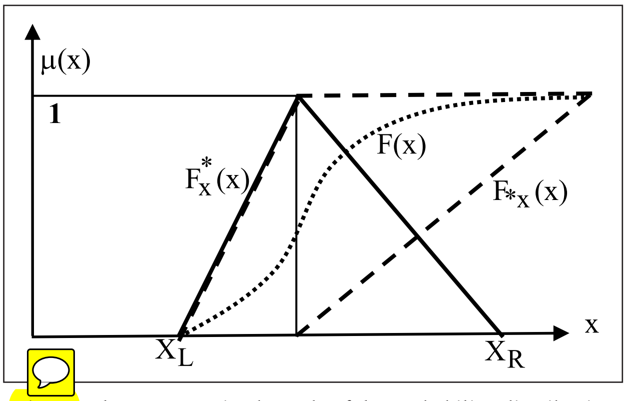

Fig. 2 : The conservative bounds of the probability distribution

under transformation. The principle allows one to use all the available information without unwittingly adding any information that is not contained in the evidence. Any random variable having distribution function defined by two parameters can be obtained by such transformation. However, the choice of the distribution should be such as to minimize the loss of information. The maximum entropy principle is mathematically an optimization problem where one seeks a *pdf* which maximizes the entropy function. If the distribution is defined by two parameters, the Gaussian distribution is obtained by this criterion (Puig and Akian, 2004). Thus, the normal distribution satisfies most conservatively the consistency condition that the probability must be less than or equal to possibility as it maximizes the entropy. The concept has been successfully applied to update uncertain parameters in various structural engineering problems (Haldar and Reddy 1992, Rahman and Zahaby 1997, Zhenyu and Chen 2002). The reliability analysis of *HUS* structures characterized by mixed probabilistic and fuzzy parameters are demonstrated in probabilistic format (Chakraborty and Sam 2007) and also in possibilistic format (Chakraborty and Sam 2011). The probability of failure obtained by such transformation approach is found to be in conformity with the failure probability bounds derived based on the evidence theory and could be a preferred approach of transformation when no specific judgment is available regarding the distribution pattern of the uncertain variables. Anoop et al. (2006) presented three approaches for converting probabilistic information, represented by a probability distribution, into an equivalent fuzzy set. The first approach is based on the method of least-square curve fitting, the second approach is based on the conservation of uncertainty (represented by the entropy) associated with the probabilistic fuzzy set in a mean square sense, and the third approach is based on the minimisation of Hausdorff distance (HD) between the probabilistic and the equivalent fuzzy sets. The effectiveness of these approaches in preserving the entropy as well as in preserving the elements of the fuzzy set and their corresponding grades of membership are also discussed with the help of a numerical example of obtaining equivalent fuzzy set for peak ground acceleration. It is found that the approach based on minimisation of Hausdorff distance provides a simple and efficient way for converting the probabilistic information into an equivalent fuzzy set. Marano et al. (2008) proposed fuzzy time-dependent reliability analysis of RC beams subjected to pitting corrosion in presence of probabilistic and non-probabilistic system parameters. The non-probabilistic parameters have been treated as fuzzy variables and probabilistic parameters have been mutated into equivalent fuzzy variables through information entropy concept.

The least conservative principle is used by Chen et al. (1998), Sophie (2000) to construct a possibility distribution from known probability distribution. It states that among all the transformation that yield possibility distribution consistent with a given probability distribution, the one that results in the minimum loss of information is the best. The transformation is obtained based on the hypothesis that the possibility of any event is greater than or equal to one i.e. it is consistent with the probability distribution. Based on the presumptions of symmetric probability distribution, the possibility distribution is assumed to be symmetric and obtained as

$$\begin{aligned}\prod(x) &= F(x)+1-F(2x_{mod}-x),\ x < x_{mod} \\ &= F(2x_{mod}-x)+1-F(x),\ x > x_{mod}\end{aligned} \tag{4}$$

where F(x) is the *CDF* of the variable x.

A transformation approach based on the Bayesian approach is proposed by Smith *et al*. (2002) to reduce the conservatism of the possibility theory. Scaling the *mf* with respect to the area under it does the transformation. The scaling factor is obtained to satisfy the axiom that the area under the *pdf* should be unity and intuitively satisfies the consistency principle. Mouchaweh et al. (2006) proposed a variable transformation, whose specificity property varies with a parameter, for transformation from probability to possibility. The performance of such transformation is compared with some well-known transformation available in the literatures and has found interesting characteristics for diagnosis by pattern recognition (in particular for discrete case) that it can be the most informative transformation and one that best distinguishes the confused elements. An efficient methodology to provide an accurate estimate of *mf* of reliability by using transformation techniques for *mf* along with solving the convolution integral by Fast Fourier Transform (*FFT*) is presented by Adduri and Penmetsa (2009). It can handle multiple limit-state functions. Application of the methodology during preliminary design stage can provide the designer insight into the system reliability and its variation due to bounds on the fuzzy variables. Balu and Rao (2012) presented a novel solution procedure for inverse reliability problems to determine the unknown design parameters such that the prescribed reliability indices are attained in presence of mixed uncertain (both random and fuzzy) variables. The proposed computational procedure involves probability of failure computation using high dimensional model representation, transformation technique to obtain the contribution of the fuzzy variables to the convolution integral and *FFT* for solving the convolution integral, reliability index and *MPP* update. Ali and Dutta (2012) reviewed the consistency principles as proposed by Zadeh, Klir, Dubois and Prade in connection with dose assessment using Probability-Possibility transformations. Anoop et al. (2012) presented a methodology for durability-based service life estimation of reinforced concrete structural elements considering fuzzy and random uncertainties. The methodology is based on combining the vertex method of fuzzy set theory with *MCS* technique and can be applied to obtain the bounds for characteristic value of failure probability from the resulting fuzzy set for failure probability.

3.2 Safety Analysis of HUS: Random and UBB Parameters

There is a class of literatures where reliability analysis of *HUS* is dealt in to consider probabilistic (random) and UBB type uncertain parameters. Ma et al. (2004) solved the evolutionary earthquake response problem of an uncertain structure with bounded and random parameters in a unified way, where the random structure is first transformed into an equivalent deterministic one by the Gegenbauer polynomial approximation and then the evolutionary random response problem of the equivalent deterministic structure is solved by a unified approach. Adduri and Penmetsa (2007) presented a method to obtain the bounds on structural system reliability in presence of interval variables. The implicit limit-state functions are modelled by using high quality approximations to estimate the reliability of system with mixed parameters. Du (2008) proposed a unified uncertainty analysis considering the effect of mixed uncertainty and it is shown that the belief and the plausibility measures can be converted to obtain the lower and upper bound probabilities in the context of evidence theory. Luo et al. (2009) investigated the reliability assessment of structures exhibiting both stochastic and bounded uncertainties by using a sequential solution technique for the nested optimization problem under mixed description of probability and convex models.

Zhang et al. (2010) developed interval *MCS* methods for structural reliability assessment when statistical parameters of distribution functions cannot be determined precisely due to epistemic uncertainty. Uncertainties in the parameter estimates are modelled by interval bounds constructed from confidence intervals. Gao et al. (2010) presented a hybrid probabilistic and interval method for engineering problems described by a mixture of random and interval variables. A random interval moment method is proposed to calculate the mean and variance of random interval variables following perturbation theory for linear equations with random and interval variables. Gao et al. (2011) further extended the approach to obtain the lower and upper bounds of the mean and standard deviations of displacements and stresses in the framework of *MCS* method.

Jiang et al. (2011) presented two kinds of hybrid reliability models based on the reliability index approach and the performance measurement approach, in which the reliability index interval and the target performance interval are employed to evaluate the reliability degree of an uncertain structure. Random distributions are used to deal with the uncertainty, while some key parameters of the distribution functions are given variation intervals instead of precise values. Based on the multi-ellipsoid convex model description for grouped *UBB* type parameters, the mathematical definition of a non-probabilistic reliability index has been presented by Kang et al. (2011) for quantified measure of safety margin. Zhang (2012) proposed an importance sampling technique to the imprecise probability for *FEM* based structural reliability assessment under parameter uncertainties. The proposed methodology generates point samples according to the importance sampling function and does not require expensive interval analyses. The limit states are computed using deterministic FEM.

4. Summary and Observations

Over the last few decades, considerable progress has been made in the field of safety assessment of system using probability theory, statistics, decision analysis, fuzzy logic, and related methods to cope with the inevitable presence of uncertainty. The probabilistic methods of safety analysis have been developed to a stage where they are ready to be applied to engineering structures; in fact those are routinely used in many applications. Traditionally, the information for probabilistic modelling was in the form of statistical data represented by analytical or empirical distribution functions and their parameters. The combination of objective and subjective information in civil engineering situations was one of the first topics examined after the initial statements of Zadeh's fuzzy set measures and theory that dealt with the subjective uncertainty. Though, various attempts to combine the information were successful; the use by the profession has not been popular. However, a turnaround has been observed to include a much broader range of information in the uncertainty modelling. The trends in the safety analysis of various systems indicate that there is a growing interest in the modelling of uncertainties using non- probabilistic approaches. In this regard it is worth noting that the usual safety analysis algorithms normally assume all the variables to be of single type i.e. either all probabilistic or all possibilistic. But the problems arise for safety analysis of HUS defined with mixed uncertain system parameters. Available literature not only failed to suggest any one of the above approaches but also seems to be incompatible for safety evaluation of such *HUS*. Though no specific methods in existing literature seems to be suitable for safety evaluation of such system, it has been recognized that whatever may be the approach, all the uncertain parameters involved in the related performance function should be single type either random or fuzzy depending on the method of analysis one intended to perform. The literature indicates various methods of transformations of possibilistic variables to equivalent probabilistic variables or vice versa. Relying on such emerging transformation concepts, the safety analysis of *HUS* have been demonstrated in the literatures either in probabilistic or in possibilistic approach. The entropy based transformation is hinged on sound mathematical basis and the theory of expressing the uncertainty information is well established and applied in various fields of engineering. It is important to note that there is a notion that mixing of fuzzy and random variables lack transparency of the reliability analysis and one should applies the probabilistic or the possibilistic safety analysis algorithms by assuming all the variables to be of single type i.e. either all probabilistic or all possibilistic. As such this approach introduces gross assumptions at the very beginning in modelling of the system parameters. Whereas in the transformation based approach, there is always some chance to lose some of the information during transformation stage. But, such approach offers more flexibility to the designer in realistic modelling of the system. However, it is not very clear from the literature, which is more justified and it is felt to need more study on this aspect.

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Importance Measures in Risk Assessment of NPPs

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Abstract

Assessment of risk associated with the operation of any facility plays an important role in decision making regarding the safety of the facility. It also helps in planning emergency measures around the facility such that the public is not unduly exposed to hazards associated with the operation of the facility. Risk implies the quantification of the frequencies of various situations (accident sequences) arising in the facility and quantification of the consequences associated with these accident sequences. This needs lot of data. However, it is very difficult to get precise data and there are uncertainties associated with the values. These uncertainties are quantified by assigning uncertainties to the parameter values and propagating these through the structure of the fault tree. In addition, it is essential to assess the sensitivity of the system failure index (unavailability) to component unavailabilities.

Various importance measures, both local and global, are used to assess the importance of the component unvailabilites. In this paper some local importance measures like Risk Achievement Worth (RAW), Fussel-Vesseley importance (FV), Differential Importance Measure (DIM) and a golbal importance measure First order Sobol index are assessed for a system (Emergency Core Coolant Injection System) of a standardized Indian PHWR as a case study. The advantages and shortcomings of various methods are examined from the point of their applicability. It has been observed that the measure RAW is insensitive to various components in first order cutsets, while the measures DIM and FV show a variation even in first order cutsets depending on the value of the unavailability. The same trend is also shown by the first order Sobol index. It has also been found that there is no dependency between different components as the sum of all first order Sobol indices of the components sum up to unity. The study for the global sensitivity was carried out using lognormal distribution for characterising the uncertainty of the component unavailabilities. The validity of the results to the use of other distributions to represent the uncertainties of component needs to be examined. Similarly, the study needs to be extended to other system that represent a different structure function of the system failure.

Keywords: **Sensitivity analysis, sobol index, importance measures**

1. Introduction

Assessment of risk associated with the operation of any facility plays an important role in decision making regarding the safety of the facility. It also helps in planning emergency measures around the facility such that the public is not unduly exposed to hazards associated with the operation of the facility. Risk implies the quantification of (i) the frequencies of various unwanted situations (accident sequences) arising in the facility and (ii) the consequences associated with these accident sequences. Quantification of frequencies involves development of fault tree and event tree models of the facility, keeping in mind the accident sequences, and quantification of these using the software packages that are widely available. Quantification of consequences involves the evaluation of release and transport of the hazardous material from the facility and an assessment of environmental impact and effect on public health.

An important input that is required for the quantification of frequencies of accident sequences is the component failure data in addition to the system design and operational data. System design and operational data, to a great extent is available from plant. However, it is difficult to get component failure data in a precise manner from either the plant operating history, if the plant is operating, or from other sources, if the plant is at the design stage. This introduces lot of uncertainty in the results of risk analysis. It is not only essential to estimate the sensitivity of the results to the uncertainties associated with the failure data but also assess the impact of uncertainties in the component failure data on the system failure frequency values.

In this paper, various important sensitivity measures and uncertainty measures are examined. As a case study, these techniques are applied to the assessment of uncertainty of a typical safety system (emergency core cooling system of a PHWR). The advantages/ disadvantages of various indices are discussed.

2. Sensitivity and Uncertainty Measures

It is often ''the lack or sparsity of data'' which prevents the analyst/decision maker from assigning a certain value to the parameters. Uncertainty in the inputs is reflected in uncertainty in model results and predictions [2]. Various measures are defined in literature to characterize the sensitivity measures. Some among these are

(i) Risk Achievement Worth (RAW)

RAW of a component is defined as the ratio of the risk when the component is completely down to the nominal risk. Thus

$$C_i^{RAW} = \frac{R_{U_i=1}}{R} \quad (1)$$

Where

$R_{Ui=1}$ corresponds to the risk of the system when the ith component unreliability is 1 and

R_l is the risk of the system when all the components have nominal values of unreliabilities.

Here risk is measured in terms of unreliability of a system if the evaluations are done at the system level; frequency of hazard if the evaluations are done at the plant level. In case of a Nuclear power plant, the second one corresponds to core damage frequency.

(ii) Risk Reduction Worth (RRW)

RRW of a component is defined as the ratio of the nominal risk to the risk when the unreliability of the component is zero.

$$C_i^{RRW} = \frac{R}{R_{U_i=0}} \quad (2)$$

Where

$R_{Ui=0}$ corresponds to the risk of the system when the ith component unreliability is 0.

(iii) Fussel – Vessely Inspection importance

Fussel – Vessely importance of a component is defined as

$$C_i^{IFV} = \frac{p_i}{R}\frac{\partial R}{\partial p_i} \quad (3)$$

Where p_i is the unreliability of the ith component.

(iv) Differential Importance

Differential Importance Measure (DIM) can be defined as the fraction of total change in Risk that is due to a change in parameter x_i. It is given by

$$DIM_i = \frac{\delta R_i}{\delta R} \quad (4)$$

Where

$$\delta R_i = R(p_i + \delta p_i) - R \quad (5)$$

and

$$\delta R = \sum_i \delta R_i \quad (6)$$

Here $R(p_i+\delta p_i)$ corresponds to the risk when the ith component unreliability is changed by a small amount δp_i. The summation is over all components in the system. DIM is the fraction of the local change in R that is due to a change in parameter i. DIM possesses the additivity property, i.e. the DIM of a group of parameters/events is the sum of the individual DIMs of the parameters/events in the group [4]

(v) Sobol indices

The measures, discussed above, are local sensitivity measures where as Sobol indices are global sensitivity analysis (SA) measures. Indicators created for global SA purposes are called global importance measures or uncertainty importance measures to differentiate them from local importance indicators and screening methods. Global SA measures are used to determine which of the input parameters influence output the most when uncertainty in the parameters is propagated through the model. In the family of global SA indicators one can include non-parametric techniques, variance based techniques, and moment independent techniques. Variance based techniques (VBTs) explain V_R, i.e. the variance of R, in terms of the individual parameters or parameter groups. They identify the parameters that contribute to the overall uncertainty in R the most, as follows. V_R is generated by the epistemic uncertainty in the parameter values [5]. V_R can be written in terms of individual parameter and parameter group contribution as

$$V_R = \sum_i V_i + \sum_{i<j} V_{ij} + \sum_{i<j<m} V_{ijm} + \ \dots + V_{12\dots k} \tag{7}$$

where k is the number of uncertain parameters. Here V_i is defined as

$$V_i = V\big(E(R|X_i = x^*)\big) \tag{8}$$

and stands for the variance of the expectation value of R given the parameter x_i taking a fixed value x^*. Similarly the other terms on the right hand side of the equation can be defined.

The first order global sensitivity index can be defined as

$$S_1(x_i) = \frac{V_1}{V_R} \tag{9}$$

Parameters that have higher contributions to variance will have higher values of V_i and thus have higher values of S_1. S_1 can, thus, be taken as uncertainty importance measure of the parameter x_i. If all the parameters x_i are independent the sum

$$\sum_i S_1(x_i) = 1 \tag{10}$$

The contribution from all the terms involving more than one parameter is zero. This implies that the importance of all the parameters is fully contained in the sum. In general, we expect the risk metric not to be additive with respect to the parameters [6]. Therefore, there will be some interactions between the parameters. The $ST(X_i)$, defined as the sum of all effects (first and higher order) involving parameter Xi, however, are capable of giving to the analyst information on the importance of terms involving more than one parameter. Through VBTs, we are able to identify the parameters that individually or as groups contribute most to the uncertainty in R in a quantitative fashion and without stating any assumption on the type of the dependence of R on the individual parameters.

3. Case Study

As a case study, emergency core cooling system (ECCS) of a typical Pressurised Heavy Water Reactor (PHWR) has been taken up. The purpose of this system is to provide core cooling in case of loss of coolant accident arising out of a breach in the primary heat transport system of a PHWR.

3.1 Description of the System

ECCS is designed to remove the decay heat from the fuel following a loss of coolant accident and provide a means of transferring decay heat to the ultimate heat sink under all credible modes of Primary Heat Transport System (PHTS) failure. A schematic of the emergency core cooling system is shown in figure. Two different systems are employed for this purpose. One is meant for handling large and medium LOCA situations and the other for handling small LOCA. ECCS operation can be split into two distinct phases namely Emergency Coolant Injection Phase (ECI) and Emergency Coolant Recirculation (ECR) phase. In this example, only the system handling large and medium LOCA situations during injection phase is only considered. A detailed description of the system is available in ref. 7.

During the ECI phase, upon the occurrence of LOCA conditions, as sensed by the low inlet header pressure signal (pressure < 55 Kg/cm^2), signal for ECI is initiated. Depending upon whether the injection is type I (only low inlet header pressure signal is present) or type II or type III (where in differential pressure signal across an inlet header and the corresponding outlet header on the opposite side is also present) the appropriate valves are operated and the heavy water injection takes place. D_2O injection is from the pressurized heavy water accumulators. As soon as the D_2O accumulators get exhausted, as sensed by the level transmitters in these accumulators, and the PHTS pressure falling to < 32 Kg/cm^2 light water injection is initiated. This is by the pressurization of the light water tank by the gas tank connected to it. This pressurized water ruptures the rupture disk and enters the PHT circuit.

3.2 Data

For evaluating the unreliability of the system, failure data of the components of the system is also needed in addition to the system data as explained above and shown in fig. 1. Since the purpose of the study is the assessment of uncertainty at the system level based on the uncertainties of component failure

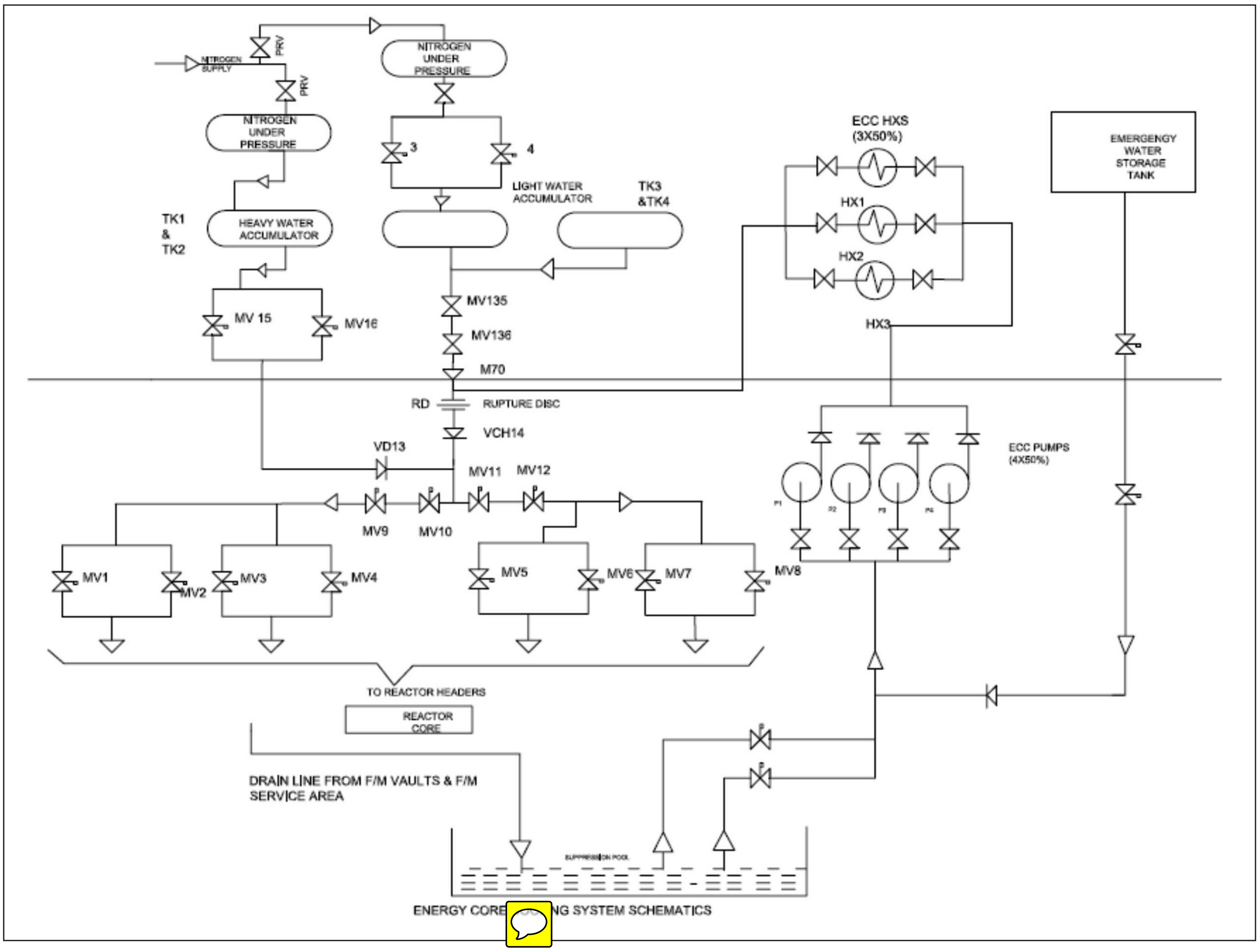

Figure 1: Simplified schematic of ECCS

data, representative data as shown in table 1 is used. The data shown in table 1 corresponds to demand failure probability of the components. Both the 5th and 95th percentile values are also included in the table in addition to the median value. Common cause failure of the instrumentation channels and motor operated valves is also considered in the analysis and a median beta factor of 0.1 is used.

Table 1: Component Failure data

Component	Median	5% value	95% value
Instrumentation channel, RA, RB, RC,INST	1.0E-4	3.3E-5	3.0E-4
Pressure Transmitter (PT)	1.0E-4	3.3E-5	3.0E-4
Pressure Indicating alarm meter (DPIA)	1.0E-4	3.3E-5	3.0E-4
Level Transmitter (LT)	1.0E-4	3.3E-5	3.0E-4
Level Switch (LIS)	1.0E-4	3.3E-5	3.0E-4
Relay ®	1.0E-4	3.3E-5	3.0E-4
Differential Pressure Transmitter (DPT)	1.0E-4	3.3E-5	3.0E-4
Check valve (VCH)	1.0E-4	3.3E-5	3.0E-4
Rupture Disk (RD)	1.0E-3	3.3E-4	3.0E-3
Motorized valve (MV)	1.0E-5	3.0E-5	3.3E-6
Actuation Logic (ACTLGC)	1.0E-5	3.3E-6	3.0E-5
β factor for MVs (Common cause) (BETAMOV)	1.0E-1	5.3E-2	2.0E-1
β factor (common cause) for Instrumentation (BETAI)	1.0E-1	5.0E-2	2.0E-1

Table 2: First 20 Minimal Cutsets of the System

Sr. No	Cutsets		Component name	Probability
1	VCH14A		Check valve	1.00E-04
2	VCH13		Check valve	1.00E-04
3	VCH13A		Check valve	1.00E-04
4	RD		Rupture Disk	1.00E-03
5	VCH14		Check valve	1.00E-04
6	VCH26		Check valve	1.00E-04
7	VCH23		Check valve	1.00E-04
8	VCHEXT		Check valve	1.00E-04
9	ACTLGC		Actuation logic	1.00E-04
10	RA	RB	Channles A & B	1.00E-08
11	MV15	MV16	Motarized valves	1.00E-10
12	MV01	MV02	Motarized valves	1.00E-10
13	BETAI	INST	CCF Instrumentation	1.00E-05
14	BETAMOV	MV	CCF MVs	1.00E-06
15	MV03	MV04	Duel failure of MVs	1.00E-10
16	MV05	MV06	Duel failure of MVs	1.00E-10
17	RC	RA	Channles C & A	1.00E-08
18	RB	RC	Channles B & C	1.00E-08
19	MV07	MV08	Duel failure of MVs	1.00E-10
20	MV09	MV10	Duel failure of MVs	1.00E-10

3.3 Uncertainty Analysis

Fault tree for the system, as presented in ref. 7 is used. The minimal cutsets for the system are generated using a c-program. It has been observed that there are 787 minimal cutsets for system failure. Among these there are 9 first order cutsets and all the remaining are second order cutsets. All the first order cutsets are shown in table 2. The first 20 minimal cutsets are shown in table 2. It can be noticed form the table that there are 9 first order cutsets. The remaining are second order cutsets. (All the remaining minmal cutsets of the system are second order cutsets.) Using the data in table 1 and assuming lognormal distribution for the parameters, Monte-carlo sampling has been carried out with 10000 trials to arrive at a mean value of the system failure probability of $1.27*10^{-3}$per demand with a variance of $3.68*10^{-9}$ (standard deviation $\sim 6*10^{-5}$). In addition, using the median values, various importance measures like DIM, RAW, FV have been evaluated and these are shown in table 3. First order Sobol indices, Sxi, for various components of the system have also been evaluated considering the impact of variability of the component failure data on system failure. These are also shown in table 3.The important contributors for the system are shown in table 3.

Table 3: Importance measures of a few components

Name	DIM	RAW	FV	Sxi
RD	0.572291	578.476	0.578054	0.926242
VCH14	0.057229	578.476	0.057753	0.010761
VCH13	0.057229	578.476	0.057753	0.010621
VCHEXT	0.057229	578.476	0.057753	0.010463
VCH26	0.057229	578.476	0.057753	0.010072
VCH14A	0.057229	578.476	0.057753	0.009629
VCH23	0.057229	578.476	0.057753	0.00931
VCH13A	0.057229	578.476	0.057753	0.008577
ACTLGC	0.005723	578.476	0.005775	0.001147
MV	0.000572	58.847	0.000578	0.000991
INST	0.005723	58.8418	0.005785	0.00106
R3870B	0.000183	2.85094	0.000185	0.001102
R6261A	0.000183	2.85094	0.000185	0.001077
LIS63C	0.000183	2.85094	0.000185	0.001043
LIS63B	0.000183	2.85094	0.000185	0.001039
DPIA52A	0.000183	2.85094	0.000185	0.001038
PT114B	0.000183	2.85094	0.000185	0.001036
R3870A	0.000183	2.85094	0.000185	0.001032
R3784C	0.000183	2.85094	0.000185	0.001028
DPI181A	0.000183	2.85094	0.000185	0.001023

4. Results and Discussion

It can be notice from table 3 that component RD (rupture disk) turns out to be the most important component irrespective of whether one uses local or global importance measures. As has been mentioned above, there are 9 first order cutsets. The local sensitivity index, RAW cannot distinguish between the various components of the first order cutsets. However, another local sensitivity measure FV, is able to distinguish between various components in first order minimal cutsets. The measure DIM also does the same. It can also be seen that the measures DIM and FV are sensitive to the value of the demand failure probability. The same trend is also indicated by the global importance measure Sx_i of the ith component. The variation in the value of the Sobol index for different components of the first order cutsets, that have the same median value of unavailability, can be attributed to the random sampling. The sum of the first order sobol indices of all the components in the system has been evaluated and it found to be very close to one. This indicates that effect of dependence between different components is either negligible or nor existent for the system.

5. Conclusions

From the study it can be inferred that the local sensitivity indices Differential Importance measure and Fussel- Vessely Importance measure exhibit the same trend and this agrees with that indicated by the global importance measure namely the Sobol sensitivity index. This has been studied for one assumed input distribution of the component failure probabilities and one fault tree that determines system structure. This, it needs to be seen whether other distributions and other system structure functions also yield the same trend.

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Stochastic Earthquake Ground Motion Model dor East Coast Region of India

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Abstract

This main objective of this article is to develop stochastic ground motion model for east coast region of India which has experienced strong earthquakes in the past but with very limited strong motion data. The broad band velocity records obtained from IGCAR array are used to calibrate the stochastic finite fault seismological model. The source and path parameters like stress drop, kappa factor, quality factor and site amplification functions are obtained from the velocity records. The derived parameters are used in the seismological model and a large synthetic database covering all possible magnitudes and hypocentral distances is generated for the east coast region. The ground motion relations for 5 % damped spectral acceleration are obtained by regression analysis. The calibrated seismological model combined with ground motion relations can be used by engineers to estimate design ground motion in east coast region.

1. Introduction

Ground motion equation is a key component in probabilistic seismic hazard analysis (PSHA). Past experience shows that a site vibrates due to earthquakes originating anywhere in a region of about 300 km radius around the site. Thus regional properties and their local details play major roles in dictating the future seismic hazard at the site. In this context, ground motion relations have an important role in practical problems. These relations describe the average or other moments of the random hazard parameter in terms of magnitude and distance. Strong motion parameters such as peak ground acceleration (PGA), peak ground velocity (PGV) and spectral acceleration (S_a) are represented as a function of magnitude (M) and source to site distance (R) based on regional strong motion array (SMA) data. While (M, R) are the most important independent variables in the attenuation equation efforts have been made to incorporate effects of fault type (FT), directivity and soil conditions (S) also. Ground motion relations are in great demand in seismic hazard analysis due to their simple form. However, the simple form of the equation introduces errors in the curve fit and this has to be reported for any attenuation relation to be meaningful. There have been attempts in deriving the ground motion prediction equations for India. Das *et al* (2006) and Sharma *et al* (2009) derived ground motion relations from the strong motion data available for Himalayan region and northeast India. On the other hand, the available information on strong motion data in the intraplate region of peninsular India is limited. In absence of strong motion data of large earthquakes, the stochastic seismological model (Boore 1983) can be used to generate samples of synthetic ground motion time histories. However, such a model needs to be calibrated with the help of some instrumental data. The input source, wave propagation, and site parameters in the seismological model should be region specific and should be obtained from the available ground motion data in the region. With the help of seismological model where in the uncertain source and path parameters can be randomly varied, one can arrive at reliable ground motion equations. Raghukanth and Iyengar (2007) derived ground motion equations for peninsular India based on the large synthetic database generated from seismological model. The methodology of deriving ground motion equations based on stochastic seismological model has been widely used in NDMA (2011) to obtain spectral attenuation relations for various regions in India. However, these equations have to be updated with the availability of new ground motion data. The focus of this article is to derive ground motion prediction relations for east coast region in India. As the reliable ground motion data of recent events is now available from the IGCAR broad band station network, it is necessary to derive new attenuation relations for this region. Since the available instrumental data is limited, the stochastic seismological model is first calibrated to the east coast region. The source, path, and site parameters in the stochastic seismological model are derived from the ground motion data. The derived path and site parameters are further used to generate a large synthetic database. Ground motion relations for response spectra are obtained from this database by regression analysis. As the east coast region has fast economical growth and infrastructure development, these relations combined with the calibrated seismological model will be helpful to obtain design response spectra and acceleration time histories.

2. Seismotectonic Setup

The east coast region with highly populated cities is a moderate seismic region in India. This region is located in the eastern part of south India which lies in the peninsular Deccan Plateau, bounded by the Arabian Sea in the west, the Indian Ocean in the south and the Bay of Bengal in the east. The geography of the region is highly diverse, encompassing two mountain ranges, the Western and Eastern Ghats, and four important rivers the Godavari, Krishna, Tungabhadra and Cauveri. The fault map of east coast region prepared from Seismotectonic Atlas of India (GSI 2000) is shown in figure 1. The southwestern parts of Tamilnadu and Kerala are traversed by few major faults namely, the NE-SW trending Pattikadu-Kottengoi fault, Tekkadi Kodaivannalu fault and NW-SE trending Tenmalai fault. The sporadic seismic activity with low magnitude events is observed in this region. The coastal basins of Tamilnadu and Pondicherry are more active than the adjacent regions due to the presence of Cauveri, Vaigai and Palar faults. The NW trending faults like Vaigai river fault, Cauveri river fault, Tirukkovillur-Pondicherry fault and the arch shaped Palar river fault are extended from coastal region to the interior. This area is also traversed by a dense network of faults, in those NW-SE trending Amaradakki fault, Manamelkudi-Tondi fault, Rajamatam fault, SW-NE trending Attur fault, Rajamatam-Devipattinam fault, vertical trending mouth of caleroon fault are major and significant. Barring one moderate event (M_b=5.2) on 5th July 1974, the seismicity of this region appears subdued. Another small and shallow event on 3rd February 1991 located over the Vasistha-Godavari fault indicating its seismogenic nature. Southwestern parts of Karnataka, northwestern parts of Tamilnadu and some parts of Kerala constitutes of three major tectonic domains namely the western part of Dharwar Craton, the pandyan mobile belt and NW-SE trending Khondalite belt. The principal structural trend of the Dharwar craton is NNW to NS, the Pandyan belt is ENE to NE and the structural trend of the Kondalite Belt of southern Kerala is NW-SE. This terrain is traversed by a dense network of NE and NW trending lineaments. The WNW-ESE trending Moyar Shear, Cauveri Fault, Vaigai River fault, the NW-SE trending Arkavati, Sakleshpur-Bettadpur, Ottapalam-Kuttampuzha Fault, Periyar fault, Achankovil Shear, NNE-SSW trending Mettur East fault and northeasterly trending Bhavani shear are some of the major discontinuities of this terrain. The most damaging event is the Coimbatore earthquake on 8th February 1900. The seismicity of this area is not evenly spread and there are distinct clusters of events around Bangalore, east of Mysore, east of Earnakulam and south of Kottayam because of Periyar and Achankovil faults. Southeastern parts of Andhra Pradesh are traversed by a fairly dense network of medium lineaments. They show variety of trends, though ENW-WSW and WNW-ESE trends appear to be dominant. The seismicity of the region is quite striking compared to other regions of the southern Indian Shield. Approximately 70 events are recorded from the Gauribidanur array. The strongest earthquake with magnitude 5.2 was recorded in this region on 27th March 1967, with its epicenter close to Ongole town.

3. Ground Motion Data

A total of six broad band stations including a central receiving station is installed and maintained by IGCAR. Only the central station at IGCAR, Kalpakkam has a strong motion accelerometer and other stations have only broad band velocity seismometers. A total of 13 local earthquakes and 291 teleseismic events from 2008-2010 have been recorded at these six stations. The epicenter and focal depth of the 13 local events which have been used for calibrating the seismological model are reported in Table 1. The minimum magnitude recorded was M_W 2 and teleseismic events up to M_w 8.4 have been recorded at these stations. The epicentres of these events along with their magnitudes are shown in Figure 1. The locations of the broad band stations are also shown in figure 1. Three component velocity time histories for two local events are shown in Figure 2 and 3. In Figure 4 the available data for east coast region is shown as a function of magnitude and epicentral distance. This brings out the existing deficiency in the database from the engineering point of view. Ideally, multiple strong motion accelerogram (SMA) data from the same event should be available for distances varying from zero to 300 km. In addition, magnitude values ranging from 4 to 8 should be covered at reasonable increments. This data set is not sufficient to develop ground motion relations. In such situations, seismological model is an alternative for generating synthetic ground motion time histories.

4. Stochastic Seismological Model

In regions lacking strong motion data, seismological models (Boore 1983) are viable alternatives and are used worldwide for deriving attenuation relationships (Atkinson and Boore 2006, Toro *et al* 1997, Iyengar and Raghukanth 2004, Raghukanth and Iyengar 2007, Raghukanth and Somala 2009). Stochastic seismological model originally proposed by Hanks and McGuire (1981) and later generalized by Boore (1983) is an alternate for simulating synthetic acceleration time histories with few known source and medium parameters. The main advantage of seismological model over other empirical approaches is that various source and path terms have been separated and the resulting ground motion is simply the product of these terms. To account for spatial and temporal variability of the slip acting on the fault plane, point source seismological model has been further improved by Beresenev and Atkinson (1997). The ground motion simulation procedure essentially consists of dividing the fault into N number of subfaults and each subfault is represented as a point source. Ground motion time histories are calculated for all point sources by the stochastic seismological model of Boore (1983). These simulated synthetic time histories are summed up by applying time lags to account for the rupture propagation. The main disadvantage of this stochastic finite fault approach is that the final results depend on the selected subfault size. To circumvent this difficulty, recently Motazedian and Atkinson (2005) introduced the concept of dynamic corner frequency and pulsing subfaults in stochastic finite fault simulation. In this approach, the Fourier amplitude spectrum of ground motion due to the s^{th} subfault at a station is derived from the point source seismological model, expressed as

$$A_s(f) = C S_s(f) e^{-\frac{\pi f r_s}{\beta Q}} G F(f) P_s(f) \tag{1}$$

where C is a constant, S(*f*) is the source spectral function, $P_s(f)$ is a filter function, G refers to the geometric attenuation, r_s is the subfault distance from the station, β is the shear wave velocity and Q is the quality factor of the region. The term F(*f*) is the site amplification which takes care of the rapid spectral decay at high frequencies (Anderson and Hough, 1984). In the present article, for the source, the single corner frequency model of Brune (1970) is used expressed as

$$S_s(f) = M_{0s} H_s \frac{(2\pi f)^2}{1+\left(\frac{f}{f_{0s}(t)}\right)^2} \tag{2}$$

M_{0s} is the moment of the s^{th} subfault, H_s is a scaling factor for conserving the energy of high-frequency spectral level of subfaults and $f_{0s}(t)$ is the dynamic corner frequency (Motazedian and Atkinson, 2005). The dynamic corner frequency, the seismic moment and the stress drop $\Delta\sigma$ are related through

$$f_{0s}(t) = 4.9\text{x}10^6 (N_R(t))^{-1/3} N^{1/3} \beta \left(\frac{\Delta\sigma}{M_0}\right)^{1/3} \tag{3}$$

where $N_R(t)$ is the cumulative number of ruptured subfaults at time *t*. The scaling factor for s^{th} sub-fault, H_s is given by (Boore 2009)

$$H_s = \sqrt{N}\frac{f_0^2}{f_{0s}^2} \tag{4}$$

Where f_0 is the corner frequency at the end of the rupture, which can be obtained by substituting $N_R(t)$ = N in equation 3. In the above equation, summation is done from 0 to the highest frequency present in the signal. To account for realistic model of earthquake rupture, Motazedian and Atkinson (2005) introduced the concept of pulsing area where the cumulative number of active subfaults, $N_R(t)$ (i.e. in equation 3) increases with time at the initiation of rupture, then becomes constant at some fixed percentage of the total rupture area referred to as percentage pulsing area. This parameter determines the number of active subfaults during the rupture of s^{th} subfault. These many subfaults are used in computing the dynamic corner frequency in equation 3.The constant C is

$$C = \frac{< R_{\theta\phi} > \sqrt{2}}{4\pi\rho\beta^3} \tag{5}$$

where $<R_{\theta\varphi}>$ is the radiation coefficient averaged over an appropriate range of azimuths and take-off angles and ρ is the density of the crust at the focal depth. The coefficient $\sqrt{2}$ in the above equation arises as the product of the free surface amplification and partitioning of energy in orthogonal directions. The filter function $P_s(f)$ is taken as

$$P_s(f) = \frac{\sqrt{N}}{H_s}\frac{1+(f/f_{0s})^2}{1+(f/f_0)^2} \tag{6}$$

The moment of s^{th} subfault is computed from the slip distribution as follows

$$M_{0s} = \frac{M_0 D_s}{\sum_{s=1}^{N} D_s} \tag{7}$$

Where D_s is the average final slip acting on the s^{th} subfault. M_0 is the total seismic moment of all the subfaults. The finite fault seismological model can be implemented in the time domain through computer simulation, consisting of three steps (Boore 1983). First, a Gaussian stationary white noise sample of length equal to the strong motion duration (Boore and Atkinson 1987) is simulated (Eq.8).

$$T = 1/f_c + 0.05r \tag{8}$$

Second, this sample is multiplied by the modulating function of Saragoni and Hart (1974) to introduce non-stationarity and then Fourier transformed into the frequency domain. This Fourier spectrum is normalized by its root-mean-square value and multiplied by the terms of equation 2, derived from the seismological model. Third, the resulting function is transformed back into the time domain, to get a sample of subfault acceleration time history. This way acceleration time histories are simulated for all the subfaults and are summed with a time delay Δt_s, to obtain the ground motion acceleration A(t), from the entire fault as

$$A(t) = \sum_{s=1}^{N} A_s(t + \Delta t_s) \tag{9}$$

It can be observed that, once the initial source and medium parameters are known, finding the ground motion by stochastic finite-fault simulation is a matter of detailed computation. The above is a general finite source model expressed in the frequency domain, valid for any region if only the various controlling parameters can be selected suitably. Here lies strength of this approach since almost all required parameters for east coast region can be worked out with the recorded ground motion data of local events.

The shear-wave velocity and density of the medium can be obtained from regional velocity models. The geometrical attenuation term G in equation (1) in the Indian shield is $1/R$ for $R < 100$ km and equal to $1/(10\sqrt{R})$ for $R > 100$ km (Singh *et al.* 1999). The shear wave velocity and density at the focal depth are fixed at 3.6 km/s and 2900 kg/m^3 respectively corresponding to compressed hard granite (Singh *et al* 1999; Mitra *et al* 2005). It can be observed from equation (1) that once the stress drop, site amplification, and Q values are known, one can simulate acceleration spectrum for any combination of magnitude and hypocentral distance. Thus, the problem of finding a region specific seismological source model reduces to the determination of Quality factor, stress drop, and site amplification such that the simulated Response spectra are compatible with the recorded data.

5. Model Parameters in Seismological Model

Among the input parameters in the seismological model, stress drop ($\Delta\sigma$), focal depth, dip, Kappa factor (κ_0) and Quality factor (Q) will vary depending on the seismo-tectonic setup and geology of the region. Thus these five parameters have to derive from the regional strong motion data.

5.1 Soil Amplification

The quality factor (equation 2) represents a part of the total attenuation of Fourier spectral amplitudes reaching the ground surface. It is well known that, the soil layers at a given site influence the final surface ground motion. The amplitude and frequency content of the bedrock motion gets modified due to the effect of local soil conditions. Although this modification is a part of the path effect, local site conditions are considered independent of the distance between the source and the site. For this reason, it becomes convenient to separate the amplification term [$F(f)$] in equation 1 in to further site and path effects as (Boore 1983)

$$F(f) = Y(f)e^{-\pi\kappa_0 f} \tag{10}$$

Where $Y(f)$ represents site amplification due to propagation of earthquake waves from the source region, where the shear-wave velocity is about 3.6 km/sec, toward the surface, where the average shear-wave velocity may be less than 760 m/sec. κ_0 is the kappa factor, to reduce the high-frequency amplitudes above some threshold frequency and characterizes the near-surface attenuation (Anderson and Hough, 1984).

5.2 Kappa Factor

The kappa factor for all the six stations is estimated from the broad band data of both the local and teleseismic events. It can be observed from equation 10 that the Fourier amplitudes depend linearly on κ_0 in logarithmic scale. This indicates that the value of κ_0 can be inferred from the slope of the best fit line to the Fourier spectra with frequency in log-linear scale.

Since, kappa factor influences the shape of the Fourier spectra at high frequency end of the spectrum, the straight-line is fitted between the corner frequency and the highest frequency present in strong motion data. The horizontal Fourier amplitude spectrum of acceleration for all the local events at ANP station is shown in Figure 5. It can be observed that at the slope of the Fourier amplitude spectrum changes at 10-12 Hz which is basically due to the site amplification. In this study, κ_0 is estimated from the slope of the smoothed log Fourier spectral amplitude at frequencies greater than 10 Hz, with frequency in linear scale (Raghukanth and Somala 2009). The mean and standard deviation of the obtained kappa factors for all the stations from local data are reported in Table 2. There are several studies regarding estimates of κ_0 for various parts of the world. Boore and Joyner (1997) estimated the kappa factor for generic rock sites in California in between 0.035-0.04. Motazedian (2006) derived the kappa factor as 0.029 for rock sites in Iran. In northeast India, Raghukanth and Somala (2009) have obtained the kappa factors for soft rock and stiff soil sites as 0.033 and 0.041 respectively. The kappa factors strongly depend on the local site condition and one can infer the type of soil beneath the instrument. High kappa factors indicate low shear wave velocity and low kappa factors indicate rock type site condition. It can be observed that except ANP station, the broad band instrument at other five stations is located on soil sites.

5.3 Horizontal-to-Vertical *(H/V)* Ratio

Several methods are available in literature for evaluating the site amplification. For a specific site, precise correction of the bedrock results is possible only when the soil section data are available, including the variation of shear wave velocity with depth. Information on shear wave velocity profiles at these six stations is not available. It has been observed that local soil effects are observed strongly on horizontal components of ground motion, but weakly on the vertical components (Nakamura 1989). The obtained kappa factors for vertical and horizontal components also support this argument (Table 2). Nakamura (1989) used a technique based on horizontal to vertical spectra ration (H/V) for evaluation of site amplification. Since amplification effects are observed strongly on the horizontal components, but weakly on the vertical one, the H/V ratio is considered to be a rough estimate of site amplification. Due to its simplicity, H/V ratio is widely used in modeling site effects in various parts of the world. (Lermo and Chavez-Garica, 1994; Theodulidis and Bard, 1995; Suzuki et al., 1995; Atkinson and Cassidy 2000; Motazedian 2006; Raghukanth and Somala 2009). In this article, the amplification function $Y(f)$ in equation 10 is estimated from H/V ratio. The overall site amplification function is obtained by multiplying the H/V ratio with the near-surface attenuation ($e^{-\pi\kappa v f}$) term. Since a portion of the kappa value will be already included in the H/V ratio, the kappa factor computed for the vertical component is used in equation 12 for computing the site response. The overall site amplification functions have been estimated at all the 6 stations using the broad band data of the thirteen events. These are presented in figure 6. Because at several stations an ensemble of ground motion records is available from the events, the mean site amplification function also has been estimated and is shown in figure 6.

5.4 Quality Factor

It is well known that the quality factor, that characterizes the anelastic attenuation of the medium, varies from place to place, depending on the seismotectonic features (Aki 1980; Jin and Aki 1988). The Quality factor increases with frequency as $Q=Q_0 f^n$ where the scaling constant Q_0 characterizes heterogeneities in the medium, and n, is related to the seismic activity in the region. Generally, a low Q_0 and high n value is observed in seismically active regions such as Himalaya, Kutch and Koyna regions in India (Mandal and Rastogi, 1998; Kumar *et al* 2005). On the other hand, stable regions like Peninsular India show a high Q_0 and low n value (Rao *et al* 1998; Singh *et al,* 2004). The Q-value determines the shape of the Fourier amplitude spectrum at high frequencies. In the present study, quality factors are determined for the east coast region by regression analysis of the recorded Fourier amplitude spectra of the thirteen events reported in Table 1. Taking logarithm on equation 1, we get

$$\text{Log}_{10}[A(r,f)]=\log_{10}C+\log_{10}[G]+\log_{10}[S(f)]-1.36\,fr/V_sQ(f)+\log_{10}[F(f)])+\log_{10}[P(f] \quad (11)$$

It can be observed from the above equation that, in order to determine Q-value, one requires information on site amplification [$F(f)$] and source excitation [$S(f)$] at each frequency. The filter function can be neglected for the point source seismological model. These three unknowns can be obtained simultaneously by solving the above equation. It can be observed from figure 4 that the available broad band data does not have ensemble of recordings at some sites due to all the thirteen events. In such situations, an inversion to simultaneously solve for quality factor, amplification and stress drop may lead to erroneous results. As mentioned earlier, apart from the type of site condition, no specific information regarding shear wave velocity and other material properties distribution with respect to depth exists at any one of the six broad band stations. In absence of this information, the *Q*-value can be obtained directly from the Fourier amplitudes of the vertical components by taking the site amplification term [*F(f)*] in equation 11 as unity. This under the assumption that vertical acceleration spectral amplitudes are affected less by local site amplification than the horizontal components. With this assumption, in equation 11, apart from the source term *S(f)* and Quality factor, *Q(f)*, all other quantities are known. The above functional form (equation 12) is an equation of a straight line whose intercept is given by the source term and slope by the *Q*-value. At each frequency, the quality factors are obtained by fitting the above functional form (equation 11) to the observed Fourier amplitudes. The *Q*-values obtained from vertical Fourier amplitude spectra at three hundred and sixty five different frequencies varying from 0.1 Hz to 8 Hz for events originating in east coast region are shown in figure 7. The *Q*-values can be approximated by a straight line as shown in figure 7 and given by

$$Q = 345 f^{0.63};\ \sigma(\varepsilon) = 0.27 \quad (12)$$

The obtained *Q*-value can be used for characterizing the tectonic activity in the east coast region. It has been found from several past comparative studies that, low values of Q_0 suggests (< 200) a tectonically and seismically active region, where as for seismically stable regions, high Q_0 (> 600) values are obtained. For moderate regions, Q_0 lies in between 200 and 600 (Kumar *et al.*, 2005). It can be observed that the obtained Q_0 is greater than 200 and *n* is lower than unity indicating that that seismic activity is moderate in these tectonic domains. The obtained quality factor for east coast region at different frequencies is shown along with the *Q*-values reported for peninsular and central India (Rao *et al* 1998; Singh *et al* 1999). It can be observed from the figure that, at low frequencies the quality factor of the east coast region is higher than that obtained for peninsular India and central India tectonic domains. This indicates that low frequency part of the spectrum attenuate faster than the high frequencies in the east coast region.

5.5 Stress Drop

After deriving the regional quality factor and site amplification, the only unknown parameter left in equation 1 is the stress drop to be determined individually for all the thirteen events. The total acceleration spectrum of the shear wave is described by equation 1. By substituting the expression for the corner frequency f_c (equation 3) in equation 1, Fourier acceleration spectra in terms of stress drop can be obtained for a given seismic moment. Given the stress drop, one can simulate Fourier amplitude spectrum at any site in the epicentral region. In this article, the finite source seismological model is iterated over a wide range of stress drop values and the Fourier amplitude spectra are computed at all the strong motion stations which have triggered during that particular event. The regional quality factor is taken from equation 12. The mean site amplification function at each individual station

Table 1. List of Local events recorded in the east coast Region

S. No	Date	Lat. (° N)	Long. (° E)	Depth (km)	Magnitude(M_w)	Stress Drop (Δσ bars)
1	18/03/2008	12.31	78.98	35	3	250
2	27/04/2008	13.33	79.49	10	2	300
3	13/05/2008	13.60	78.49	39	3	120
4	23/05/2008	14.34	78.06	35	3	200
5	07/06/2008	12.88	79.05	45	4	220
6	07/06/2008	12.71	79.12	5	2	200
7	06/09/2008	11.99	78.58	17	3	90
8	15/10/2008	13.04	79.60	25	3	100
9	01/11/2008	13.73	78.74	35	3	120
10	31/03/2009	12.90	80.06	24	2	280
11	02/09/2009	11.64	79.30	14	3	150
12	14/10/2009	13.45	79.63	26	3	60
13	11/12/2009	14.27	80.00	35	3	90

Table 2. Location of broad band stations and kappa values

S.No	Station	Lat. (°N)	Long. (°E)	Kappa (Horizontal component) μ ± σ	Kappa (Vertical component) μ ± σ
1	ANP	12.56	80.12	0.05±0.05	0.03±0.04
2	CPT	12.66	79.99	0.14±0.35	0.05±0.03
3	CRS	12.56	80.17	0.08±0.15	0.04±0.04
4	ILL	12.73	80.15	0.09±0.04	0.04±0.12
5	MMT	12.66	80.11	0.08±0.33	0.05±0.06
6	PLM	12.44	80.05	0.09±0.16	0.03±0.06

Table 3 Coefficients in the attenuation relation (Equation 14) for the east coast region

Period	C_1	C_2	C_3	C_4	C_5	C_6	C_7	C_8	σ(ε)
0.0000	-3.4144	1.1720	0.0028	-0.0028	-1.4224	0.0192	0.9949	0.1115	0.3705
0.0100	-3.4174	1.1703	0.0029	-0.0028	-1.4218	0.0192	0.9951	0.1114	0.3699
0.0150	-2.4284	1.0285	0.0125	-0.0029	-1.4051	0.0163	1.0129	0.1084	0.4009
0.0200	-2.3268	1.0313	0.0119	-0.0029	-1.3888	0.0147	1.0240	0.1079	0.3879
0.0300	-2.3516	1.0473	0.0104	-0.0029	-1.3686	0.0136	1.0325	0.1042	0.3733
0.0400	-2.4567	1.0633	0.0091	-0.0028	-1.3581	0.0134	1.0322	0.1032	0.3691
0.0500	-2.6093	1.0995	0.0065	-0.0028	-1.3601	0.0141	1.0257	0.1026	0.3685
0.0600	-2.8893	1.1599	0.0020	-0.0027	-1.3532	0.0154	1.0141	0.1018	0.3696
0.0750	-3.2246	1.2180	-0.0025	-0.0027	-1.3385	0.0143	1.0194	0.1008	0.3703
0.0900	-3.6000	1.3003	-0.0087	-0.0027	-1.3268	0.0145	1.0161	0.1003	0.3722
0.1000	-3.8783	1.3592	-0.0128	-0.0027	-1.3237	0.0153	1.0098	0.0995	0.3737
0.1500	-5.3508	1.7029	-0.0378	-0.0026	-1.3030	0.0205	0.9705	0.0956	0.3769
0.2000	-6.7476	2.0380	-0.0620	-0.0025	-1.2847	0.0231	0.9524	0.0930	0.3836
0.3000	-9.4013	2.7085	-0.1095	-0.0024	-1.2768	0.0357	0.8945	0.0909	0.3925
0.4000	-11.5496	3.2395	-0.1462	-0.0024	-1.2627	0.0446	0.8666	0.0912	0.3985
0.5000	-13.3484	3.6842	-0.1767	-0.0024	-1.2552	0.0503	0.8493	0.0902	0.4020
0.6000	-15.1212	4.0980	-0.2044	-0.0023	-1.2392	0.0482	0.8514	0.0884	0.4045
0.7000	-16.2935	4.3783	-0.2225	-0.0023	-1.2377	0.0515	0.8468	0.0872	0.4052
0.7500	-16.8280	4.4922	-0.2298	-0.0023	-1.2306	0.0493	0.8511	0.0862	0.4052
0.8000	-17.3899	4.6303	-0.2382	-0.0023	-1.2399	0.0537	0.8426	0.0879	0.4039
0.9000	-18.4477	4.8486	-0.2515	-0.0022	-1.2289	0.0494	0.8502	0.0865	0.4035
1.0000	-18.9602	4.9521	-0.2575	-0.0022	-1.2248	0.0474	0.8556	0.0868	0.4027
1.2000	-20.3091	5.2007	-0.2704	-0.0022	-1.2184	0.0445	0.8676	0.0873	0.3988
1.5000	-21.5313	5.4096	-0.2795	-0.0021	-1.2252	0.0433	0.8749	0.0858	0.3950
2.0000	-22.8575	5.5712	-0.2819	-0.0021	-1.2298	0.0357	0.9058	0.0888	0.3933
2.5000	-23.6093	5.5902	-0.2763	-0.0021	-1.2149	0.0240	0.9566	0.0877	0.3983
3.0000	-23.9802	5.4872	-0.2602	-0.0020	-1.2128	0.0147	1.0222	0.0877	0.4010
4.0000	-24.1363	5.2294	-0.2296	-0.0019	-1.2029	0.0059	1.1465	0.0851	0.4100

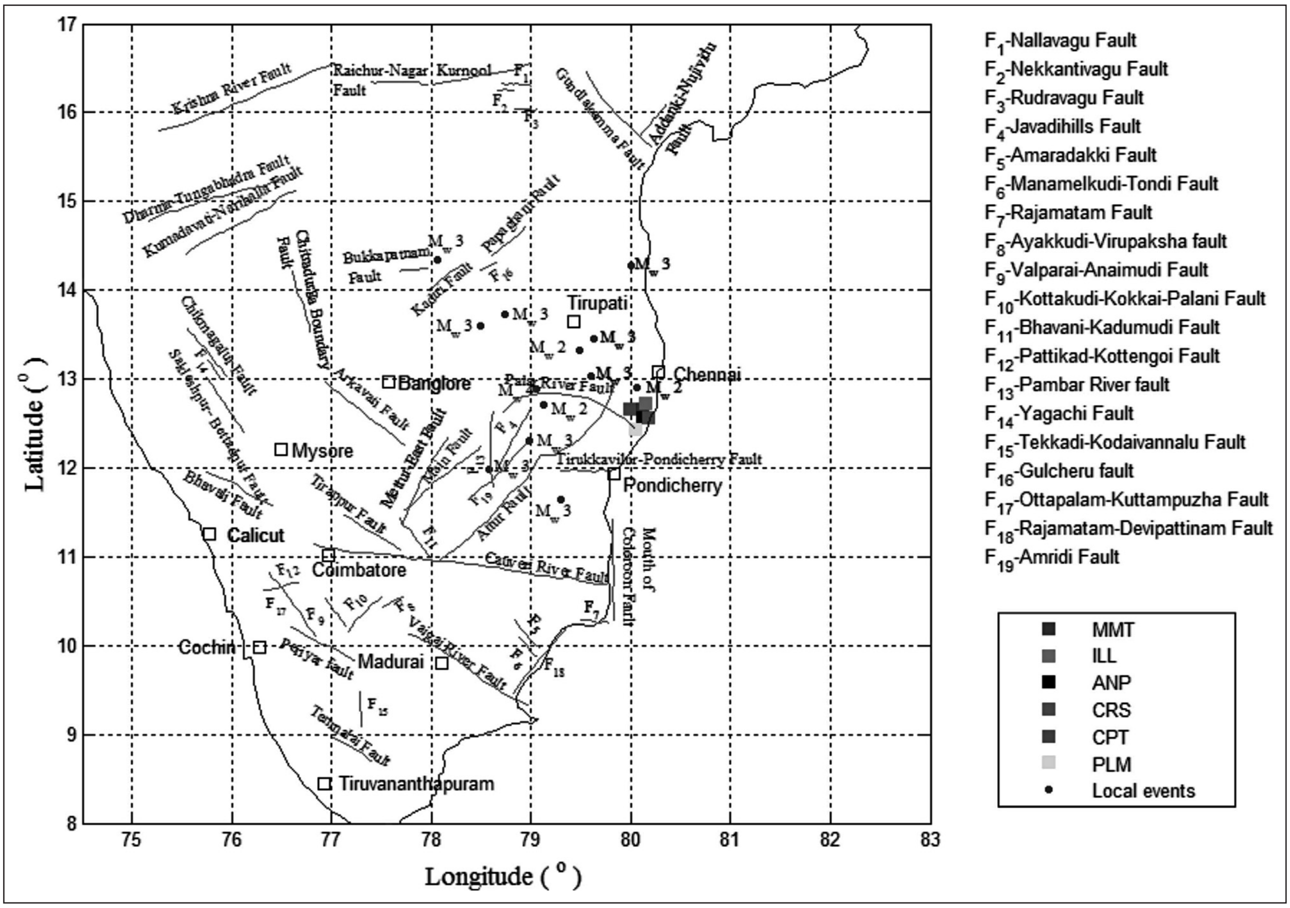

Figure 1. Fault map of south India. Filled squares - Location of broad band stations. Dots - Epicenters of local events recorded by the stations, CRS- central receiving station, IGCAR.

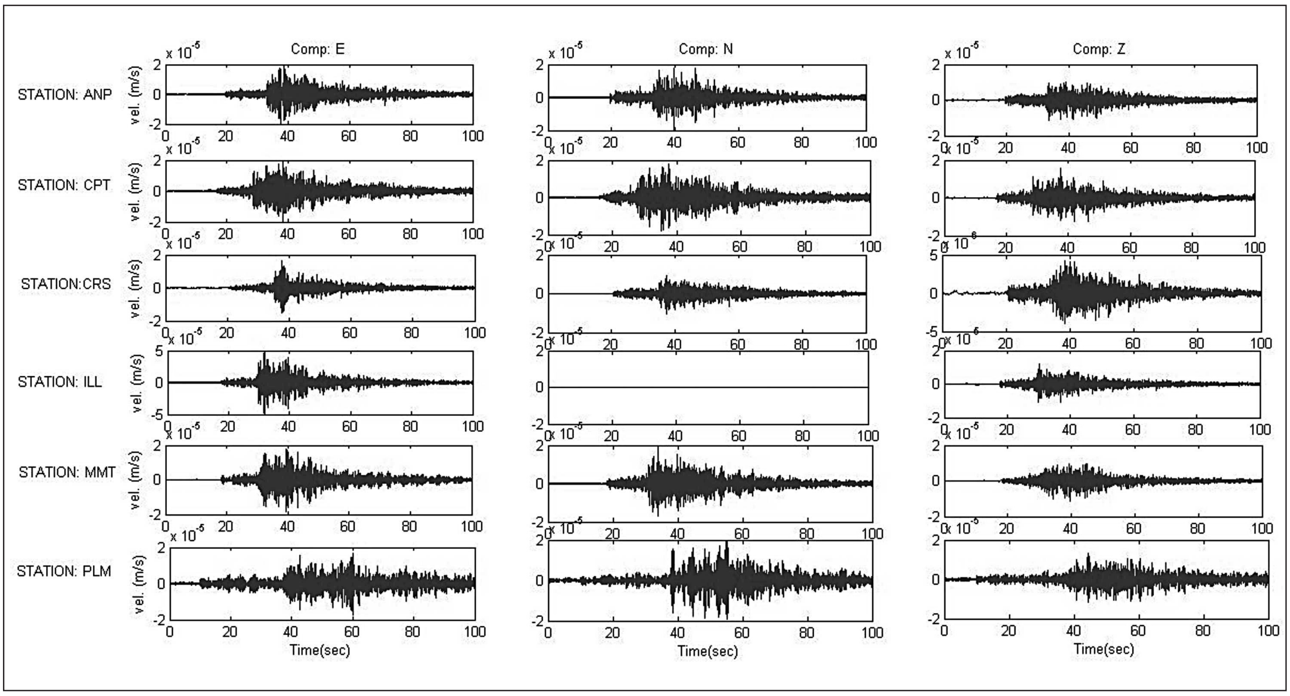

Figure 2. Recorded velocity time histories for a local event (14/10/2009) Mw 3 near Puttur, Tamilnadu

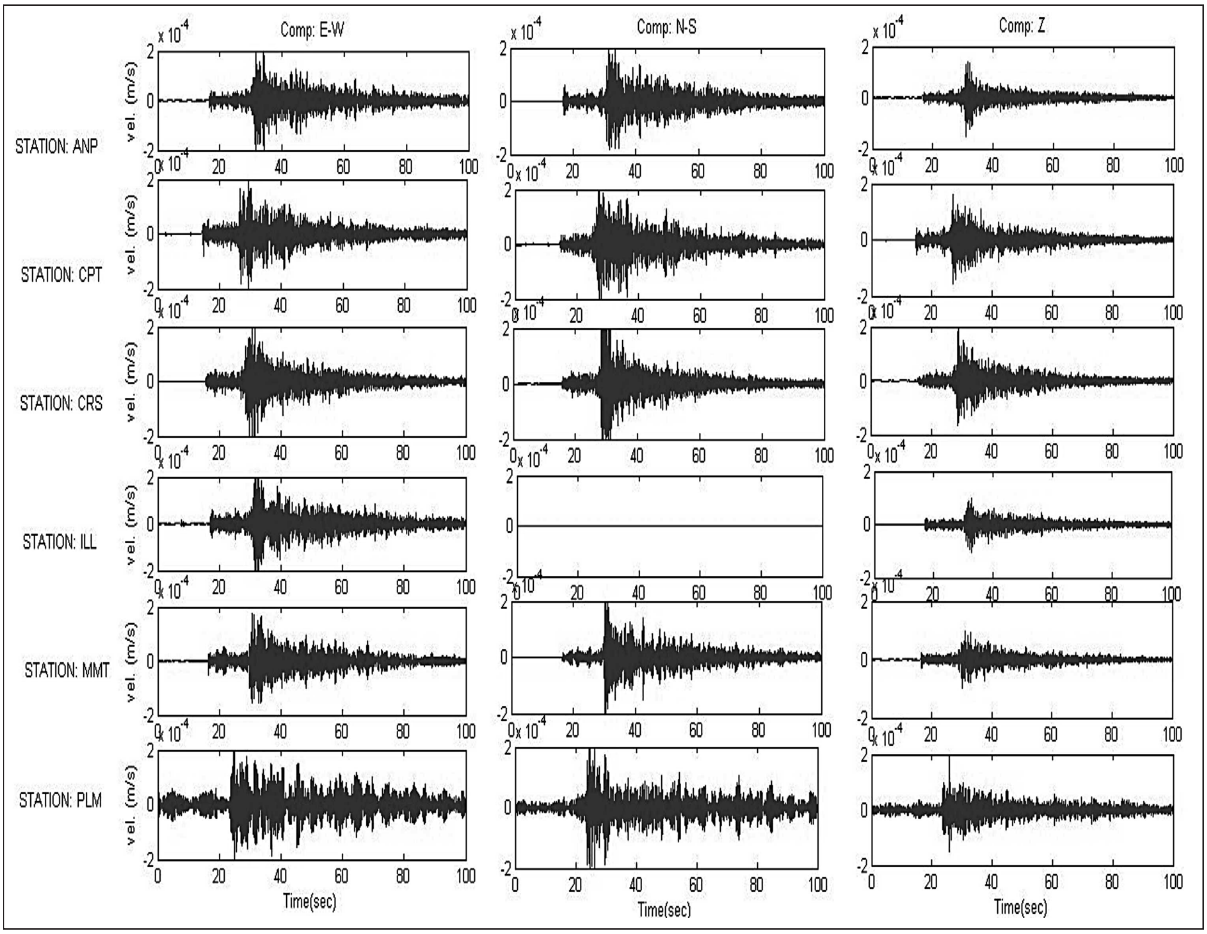

Figure 3. Recorded velocity time histories for a local event (7/6/2008) Mw 4 near Vellore, Tamilnadu

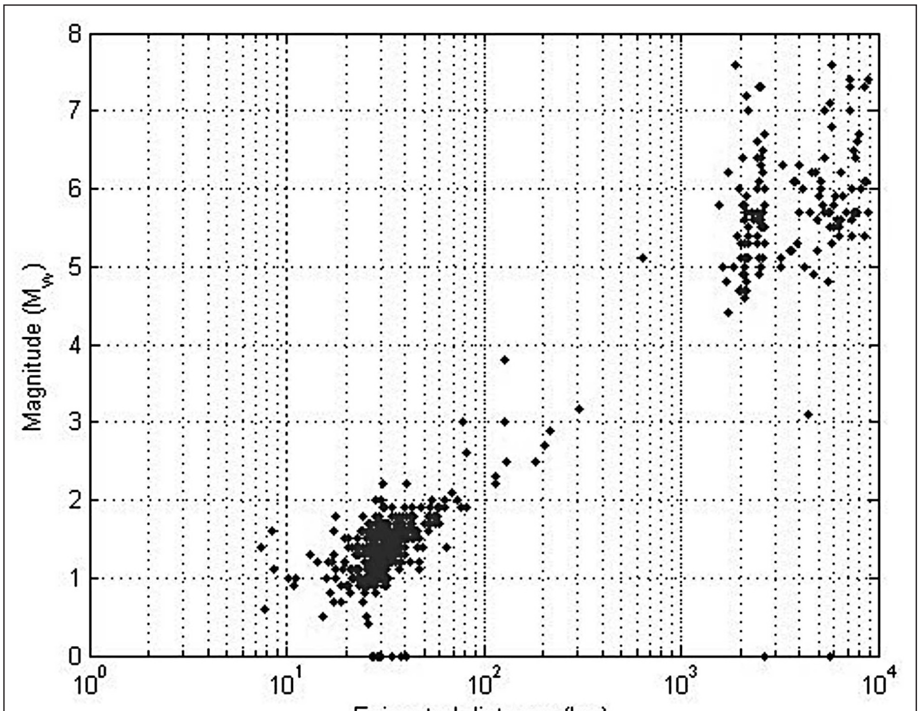

Figure 4. Available instrumental database for the east coast region

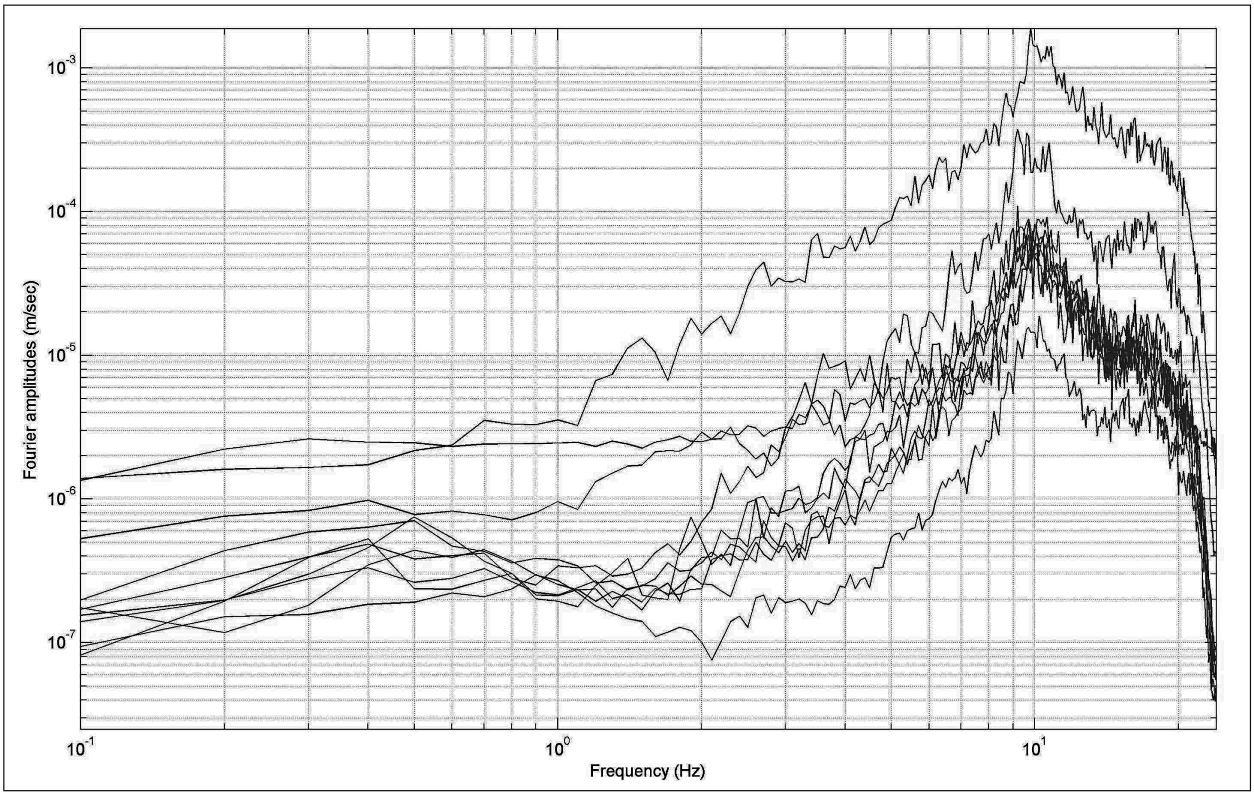

Figure 5. Fourier amplitude spectrum of horizontal components for thirteen local events (Station : ANP)

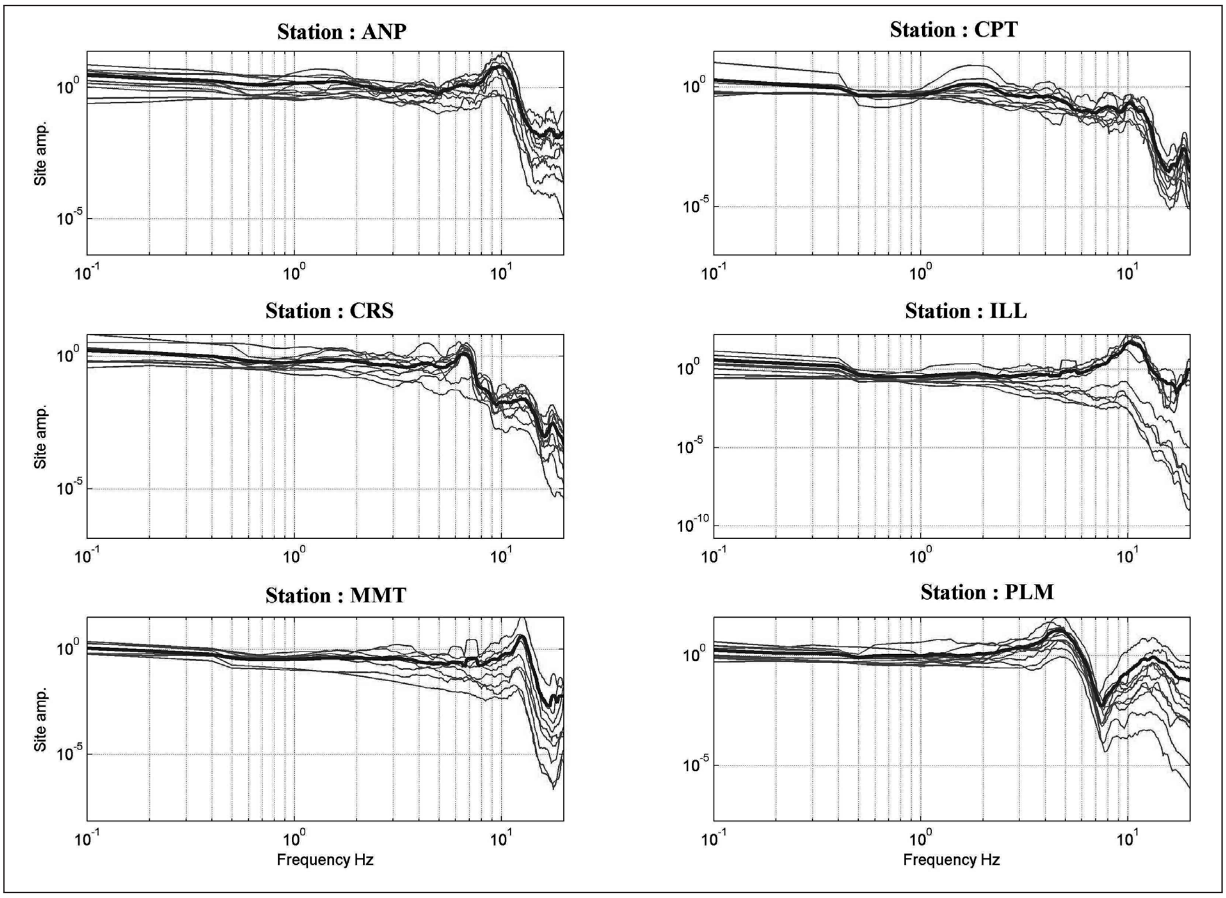

Figure 6. Site amplification functions based on the H/V ratio.

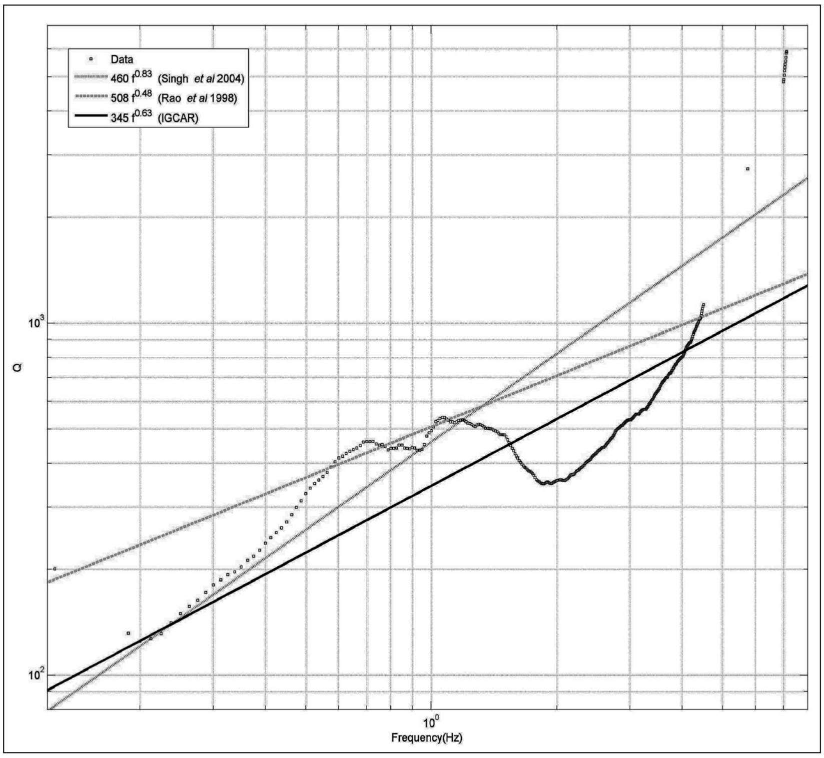

Figure 7. Regional Q-value for the east coast region

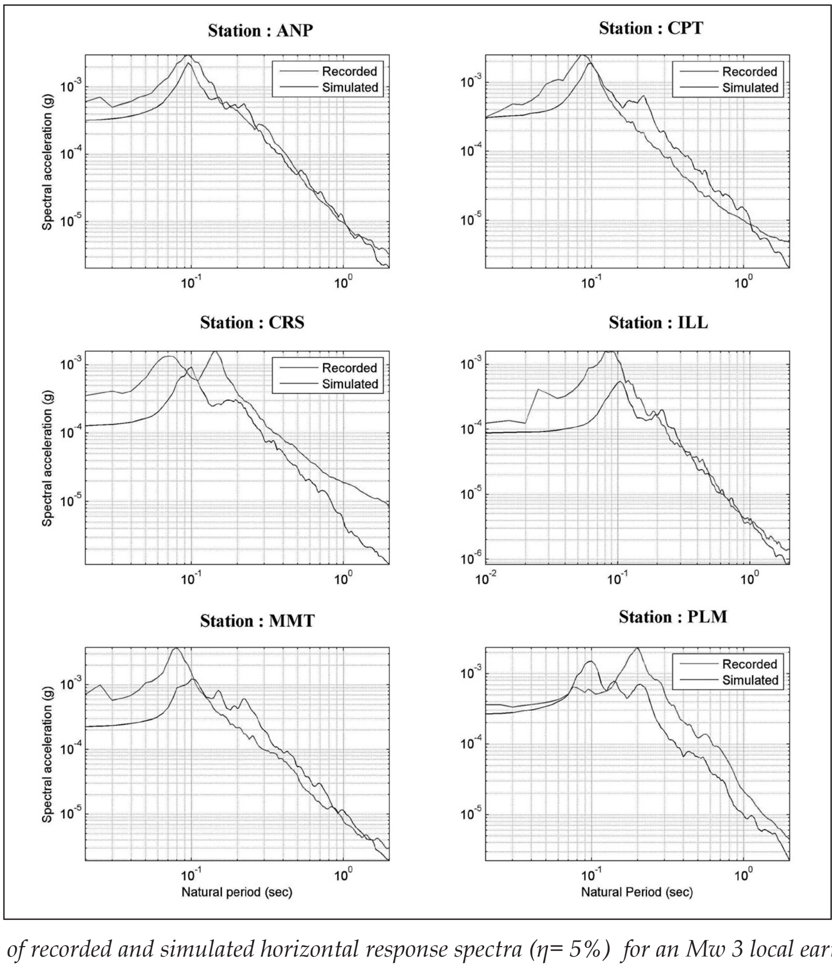

Figure 8a. Comparison of recorded and simulated horizontal response spectra (η= 5%) for an Mw 3 local earthquake (23 May 2008)

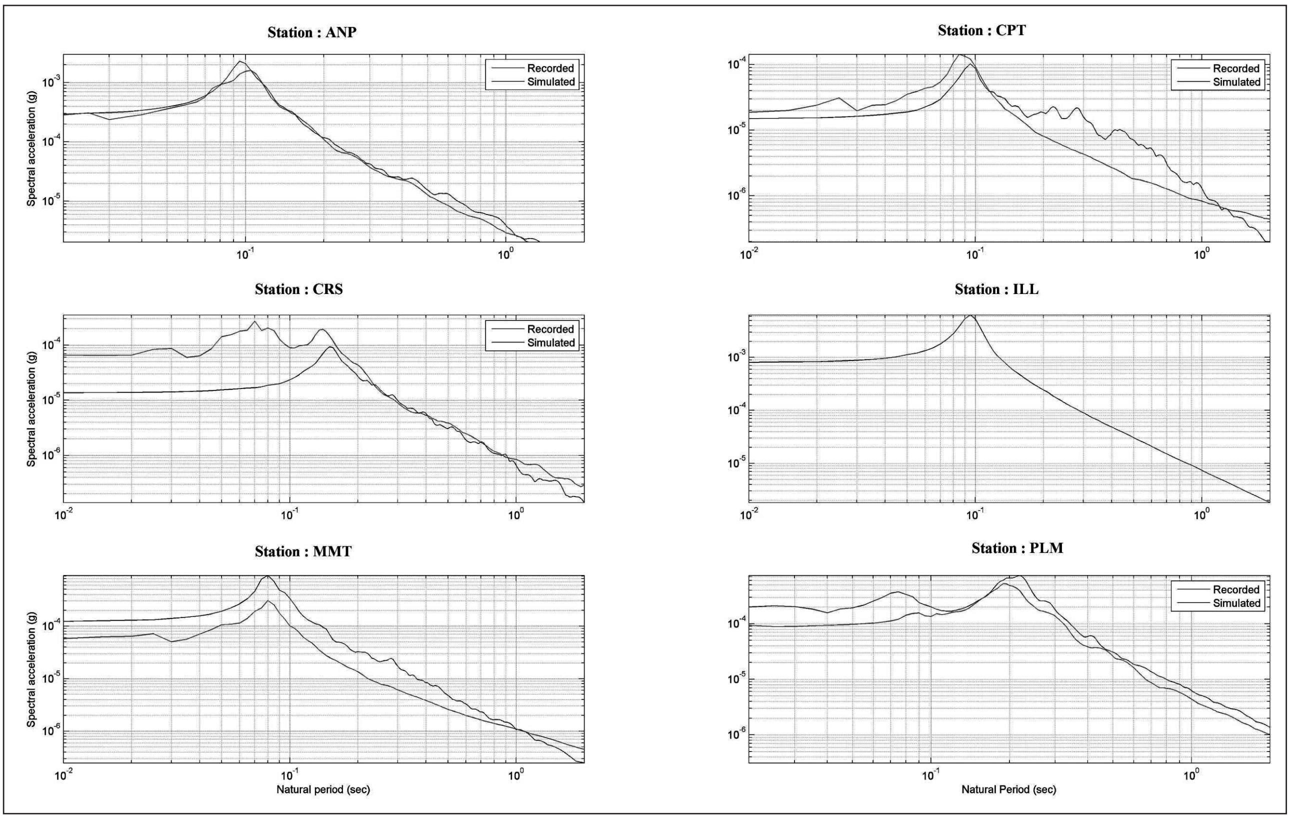

Figure 8b. Comparison of recorded and simulated horizontal response spectra (η= 5%) for an Mw 3 local earthquake (18 March 2008) .

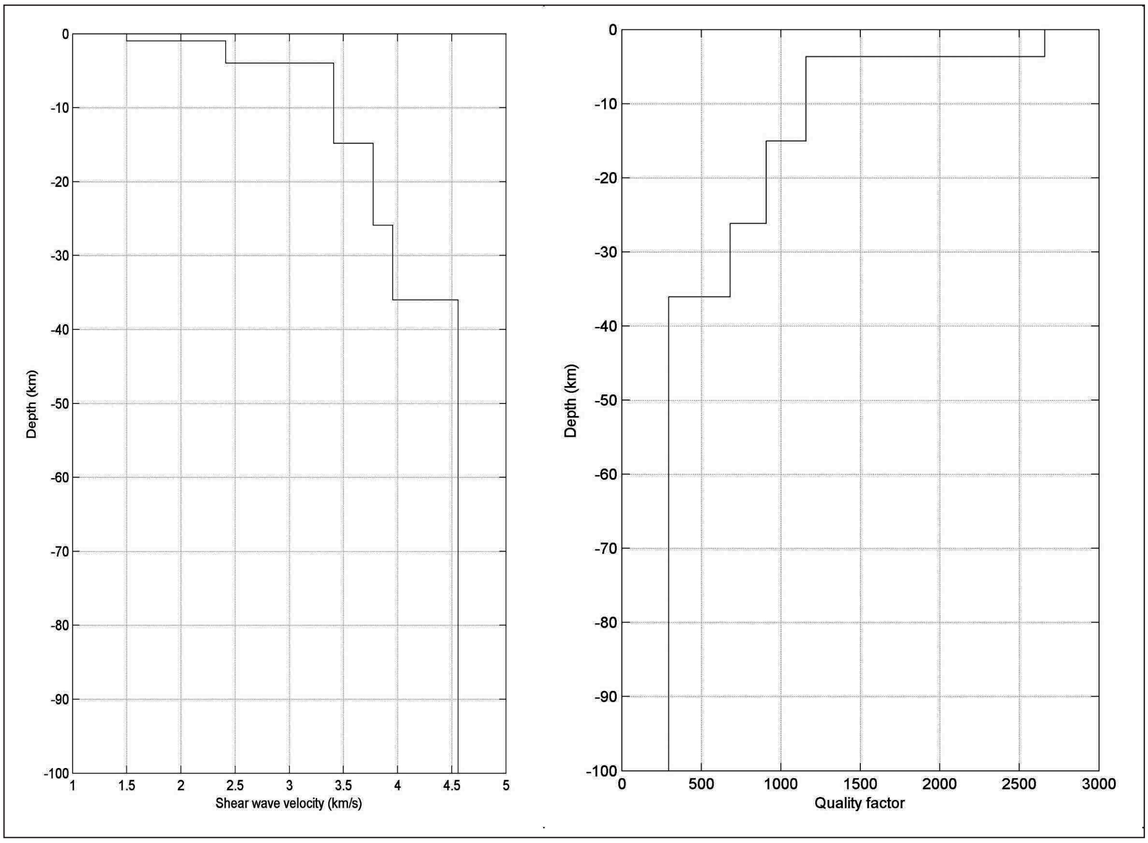

Figure 9. Shear wave velocity and quality factor profile of A-type site in east coast region (Boominathan 2004; Parvez et al 2003)

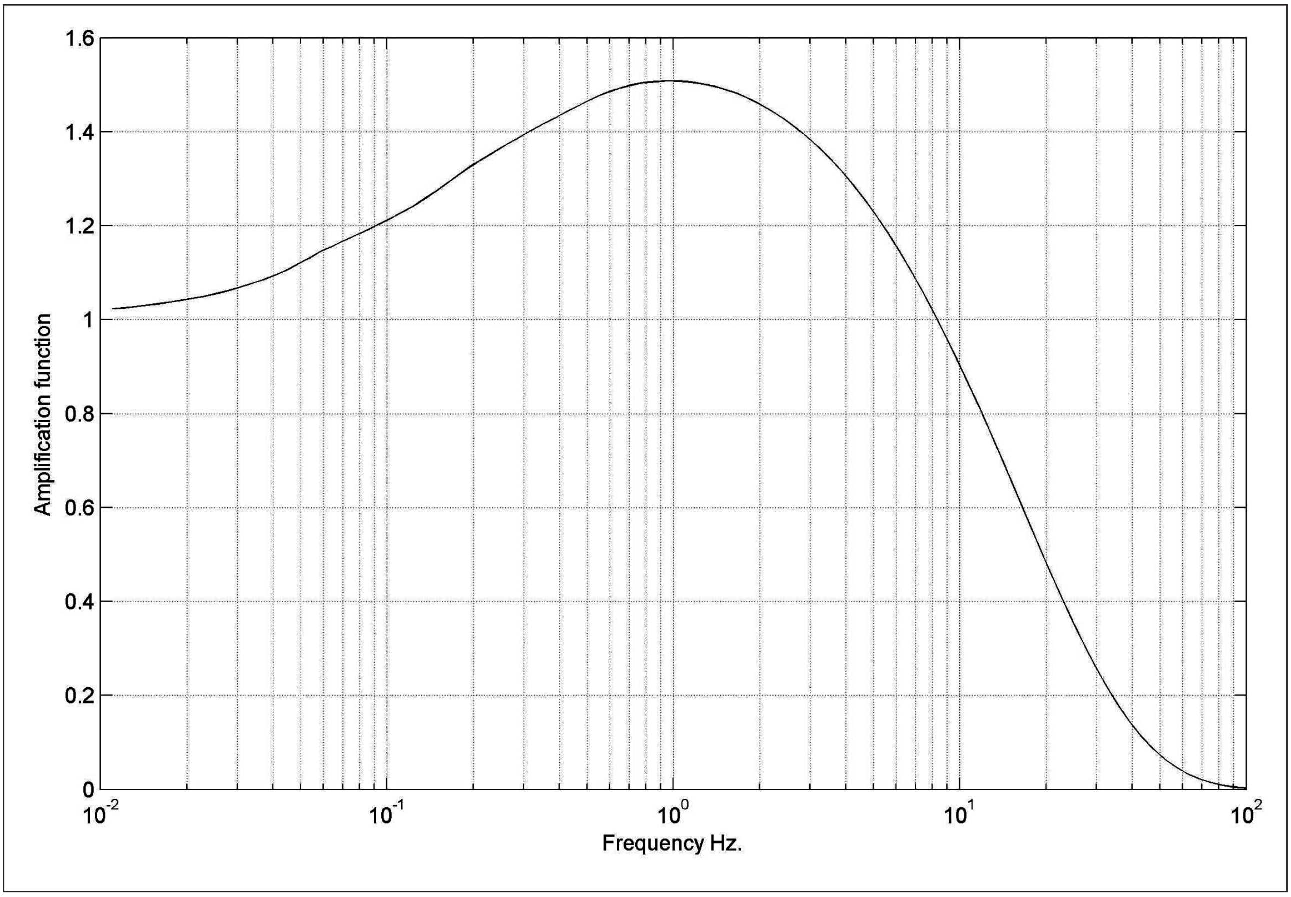

Figure 10. Crustal amplification function for A-type site

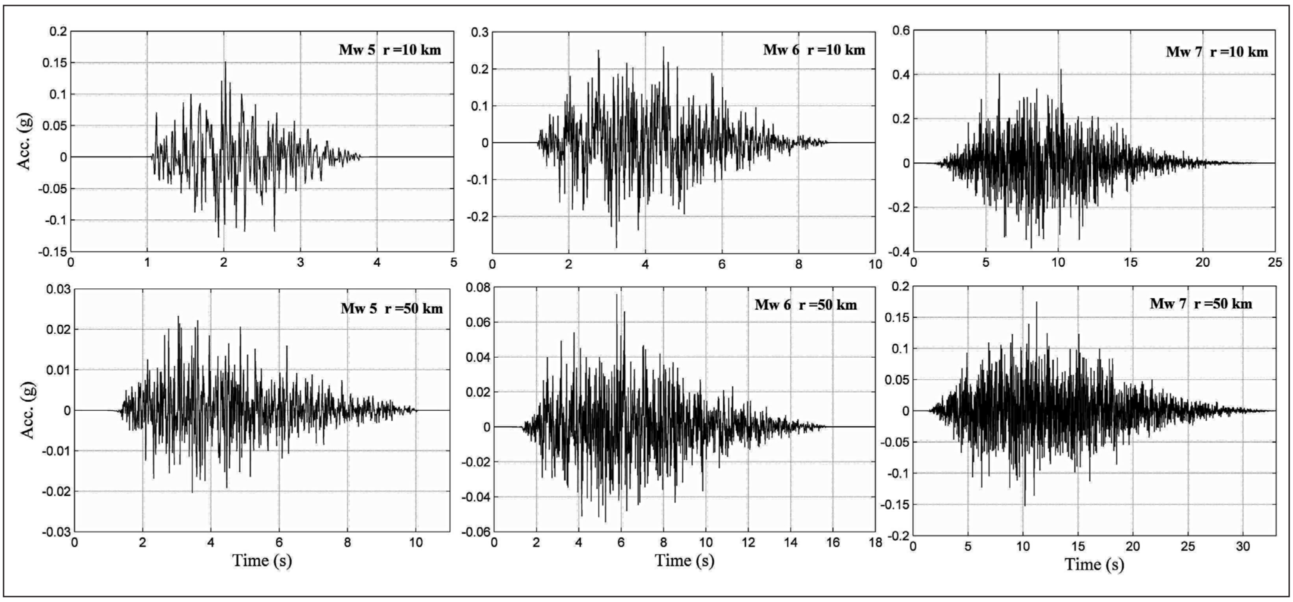

Figure 11. Simulated sample acceleration time histories

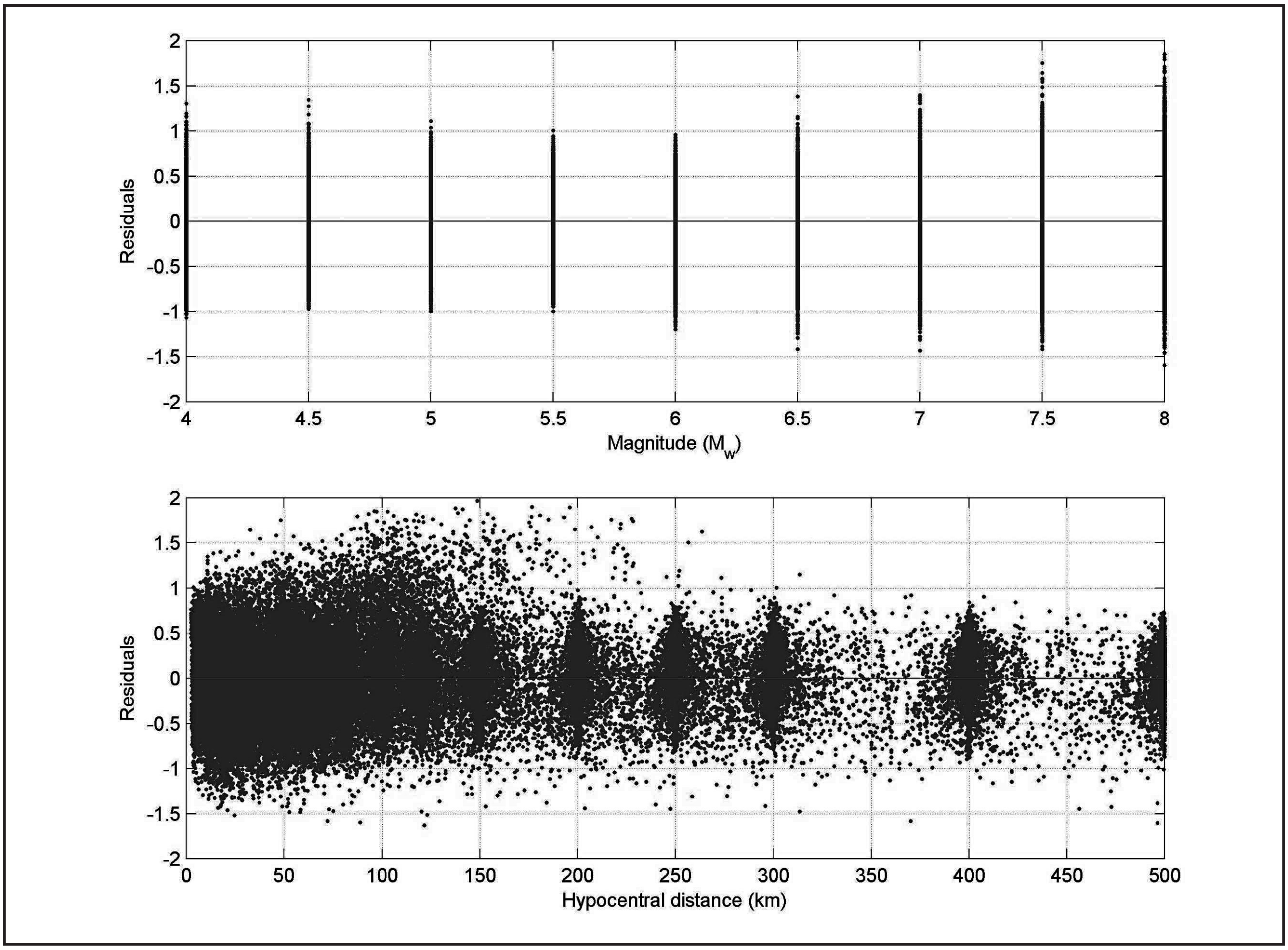

Figure 12. (log simulated PGA - log PGA by eq. 12) versus Mw and hypocentral distance for East coast Region region

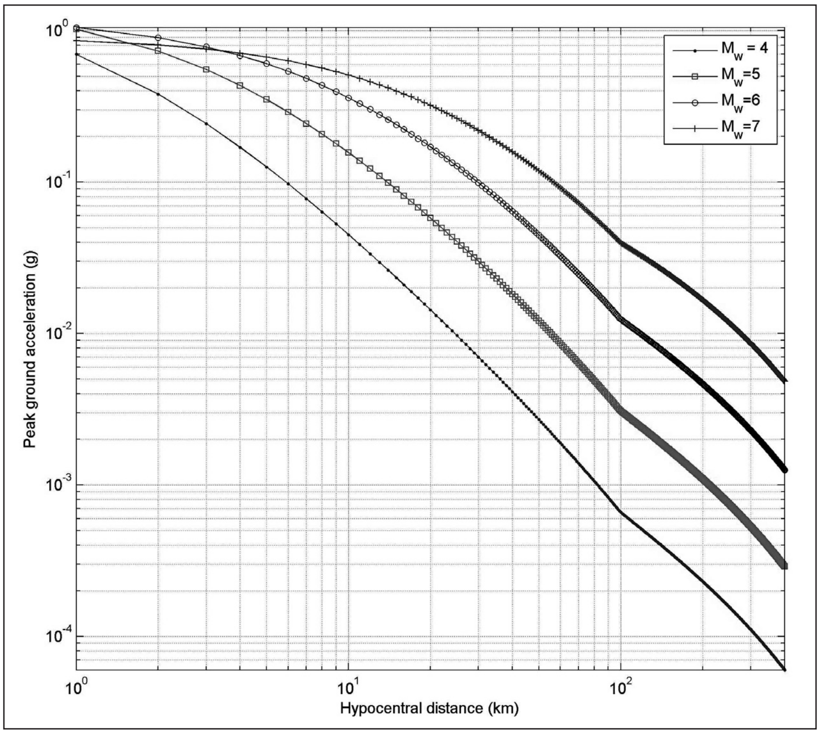

Figure 13. Attenuation of PGA with hypocentral distance

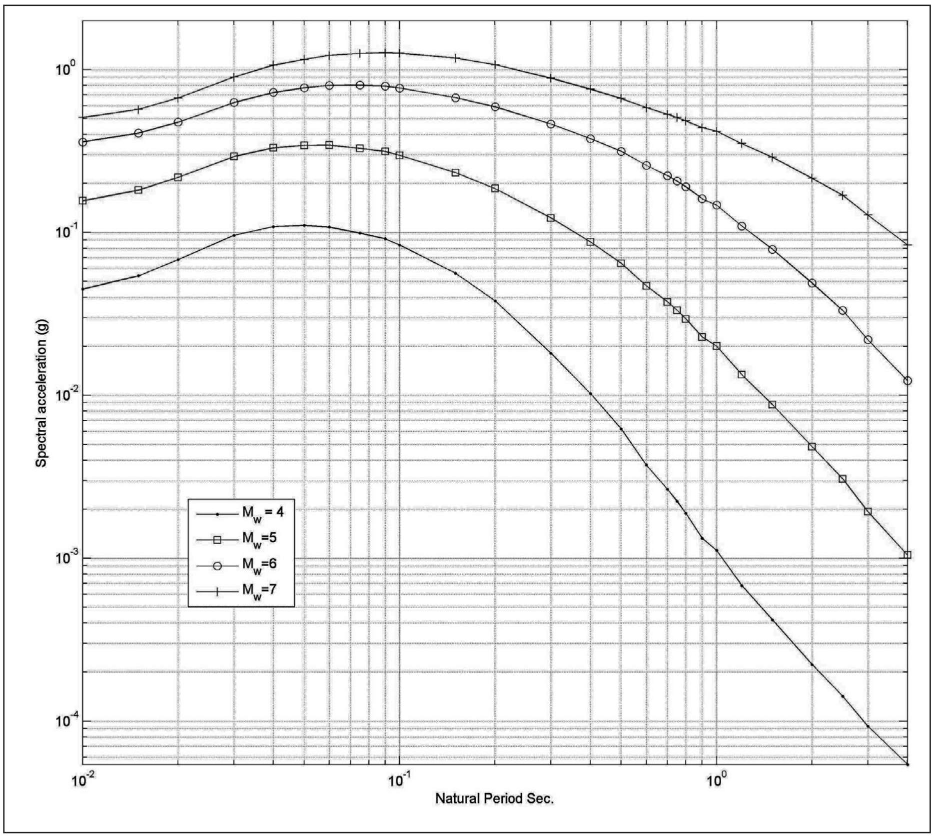

Figure 14a. Response spectra for a typical site in the east coast region (r= 10 km)

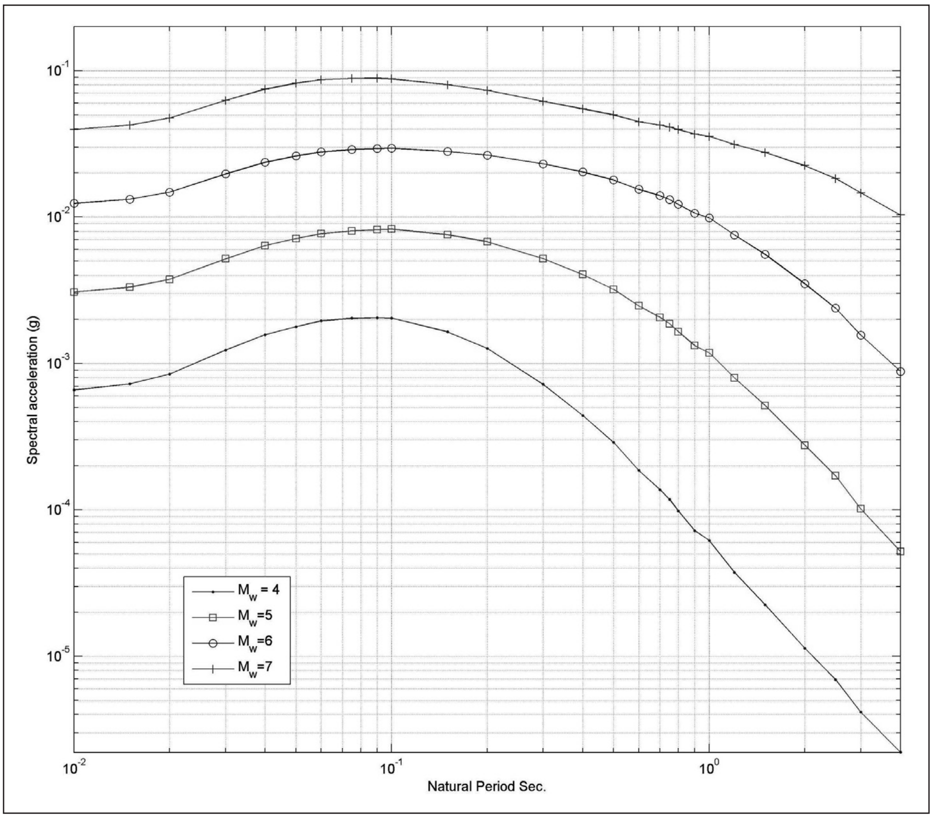

Figure 14b. Response spectra for a typical site in the east coast region (r= 100 km)

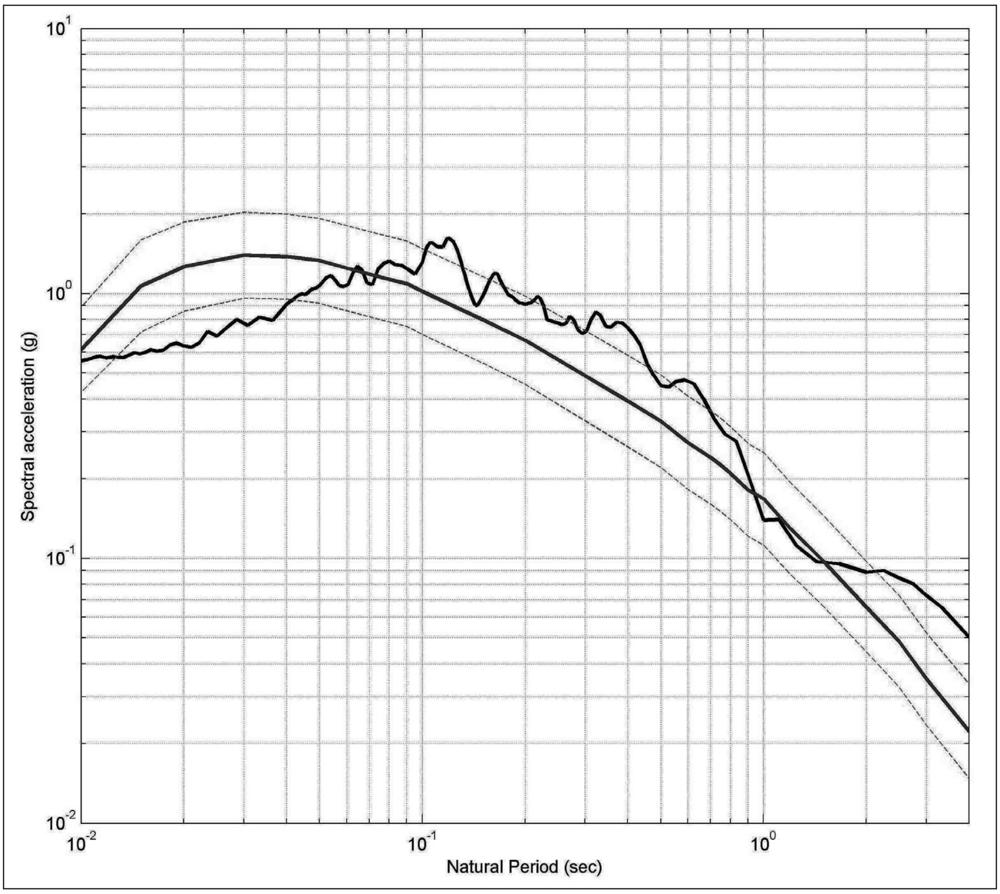

Figure 15. Comparison between estimated response spectra and spectra from data of Koyna earthquake of 11 December 1967 (Mw = 6.5, hypocentral distance = 16 km, damping = 5%)

is taken from Figure 6. The optimum stress drop is found by minimizing the mean square error between the observed and the recorded data. The error for a given event is a function of the hypocentral distance (r) and the frequency (f). The mean square error (ε^2) is defined as the value integrated over all values of f and r. By integrating over K stations and N_f frequencies, the mean square error is found out from the observed Fourier spectra [O_{ij}] and the theoretically calculated spectra [Y_{ij})] as

$$\varepsilon^2 = \sum_{j=1}^{K}\sum_{i=1}^{N_f}\left[\log(O_{ij}) - \log(Y_{ij})\right]^2 \tag{13}$$

The mean square error for different stress drop ($\Delta\sigma$) is estimated numerically for all the events. The stress drop is varied from 10-1000 bars. The obtained optimum stress drop for all the thirteen events is reported in Table 1. The stress drops of the thirteen events vary from 60 to 300 bars. In figure 8, the recorded and simulated response spectra at 5 % damping, are shown for comparison at six stations for two local events in the east coast region. Although there are differences, the correlation between the response spectra is quite good for almost all the stations. The favorable comparison between the simulated and recorded response spectra suggests that the seismic parameters obtained in this study can provide reasonable estimates of ground motion during strong events in east coast region.

The obtained source and path parameters from the broad band motion data can be used in the stochastic seismological model to simulate acceleration time histories and response spectra for any typical site in east coast region of India.

6. Synthetic Ground Motion Database

After estimating the input parameters, the finite fault seismological model is further used to simulate ground motion for various combinations of magnitudes and distances. After estimating the amplification, spectral acceleration values are simulated for moment magnitude (M_w) ranging from 4 to 8 in increments of 0.5 units, at 20 values of hypocentral distances ranging from 1 to 500 km. To capture finiteness of the source the ground motions are also simulated for eight azimuths ranging from 0^0 to 315^0 in increments of 45^0. Thus a total number of 160 distance samples are considered for each magnitude. In all, there are 1440 pairs of magnitudes and distances. The parameters in the seismological model such as focal depth, radiation coefficient, slip distribution, pulsing percentage and stress drop are treated as random variables. Based on the focal mechanism solutions of past earthquakes, the focal depth and dip angle of the rupture plane are treated as a uniform random variable. The focal depth is varied from 5km to 15 km where as dip angle is taken to vary between 10^0-80^0 (Kayal 2008). Apart from the above parameters, the S-wave radiation coefficient ($<R_{\theta\varphi}>$) varies randomly within particular intervals. Here, following Boore and Boatwright (1984), the S-wave radiation coefficient is taken to be in the interval 0.48–0.64. In addition to the above parameters the slip distribution and pulsing percentage area is also required in the simulation. The slip distribution on the rupture plane is treated as uniform for all the subfaults. The pulsing percentage area is varied from 25% - 75% (Atkinson and Boore 2006). Based on Table 1, the stress drop is taken to vary between 50-300 bars. The quality factor is taken from equation 12.

After fixing the range of source parameters, the next step is to estimate the site amplification. The average shear wave velocity in east coast region is greater than 1.5 km/s (Boominathan 2004) corresponding to a A-type hard rock site as per IBC (2009). Such sites are common in east coast region and are also met in central India. The shear wave velocity profile along with corresponding quality factor for east coast region is shown in Figure 9 (Boominathan 2004; Parvez *et al* 2003). Modification between bedrock and A-type site is a linear problem in one dimension and hence for such sites amplification can be directly found by the quarter-wavelength method of Boore and Joyner (1997). The crustal amplification function estimated for a typical A-type site in east coast region is presented in Figure 10.

Since the stress drop, focal depth, dip, radiation coefficient, pulsing percentage area are random variables, we have included the uncertainty arising out of these parameters also. Accordingly, fifty samples of these five seismic parameters are generated and these are combined using the Latin Hypercube sampling technique (Iman and Conover 1980) to select for each magnitude value fifty sets of random seismic parameters. Thus, a database of 72,000 PGA and S_a samples corresponding to 1440 simulated earthquake events are generated. This synthetic database is developed for east coast region for A-type rock site condition. The simulated aacceleration time histories for magnitudes Mw 5, 6 and 7 at hypocentral distances of 10 km and 50 km are shown in Figure 11. It can be observed that duration of acceleration time histories increases with magnitude and hypocentral distance consistent with the seimological model (Equation 8).

7. Ground Motion Relations for A-Type Rock Condition

The simulated synthetic database is used to develop ground motion prediction relations for east coast region of India. The first step in developing such empirical relations is to fix the functional form of the equation. Several functional forms are available in the literature to reflecting salient aspects of the spread of ground motion (Sadigh *et al* 1997, Toro *et al* 1997, Campbell 2003, Atkinson and Boore 2006). After reviewing the various available forms of equations, it has been decided to develop the ground motion relation for the east coast region in the form (NDMA 2011)

$$\ln\left(\frac{S_a}{g}\right) = c_1 + c_2 M + c_3 M^2 + c_4 r + c_5 \ln\left(r + c_6 e^{c_7 M}\right)$$
$$+ c_8 \log(r) f_0 + \ln(\varepsilon) \quad f_0 = \max(\ln(r/100), 0) \qquad (14)$$

Here S_a is the spectral acceleration, M is the moment magnitude, r is the hypocentral distance in kilometers. This form of the attenuation accounts for geometrical spreading, anelastic attenuation and magnitude saturation similar to the finite source seismological model discussed previously. The coefficients of the above equation are obtained from the simulated database of 72,000 samples by regression analysis. Since the equation is empirical there would be other forms that can equally well represent the data. The sufficiency of a particular equation can be verified by plotting the residuals in the goodness of fit to detect systematic trends that indicate existence of bias. In figure 12, residuals (ln ε) are plotted as function of magnitude and hypocentral distance. Absence of discernable trend verifies the sufficiency of the above equation to represent ground motion for further work. The coefficients c_1, c_2….c_8 and the standard error are shown in Table 3 as functions of natural period for the east coast region. These results can be used to construct the mean and (mean+sigma) response spectrum on A-type rock in east coast region. In Figure 13 the attenuation of PGA with hypocentral distance is shown for different magnitude values. Figure 14 shows the response spectra for magnitudes, Mw = 4, 5, 6 and 7 at two hypocentral distances of 10km and 100km. As mentioned earlier the ground motion data available for east coast region is limited. Hence, it would be interesting to see how the derived synthetic attenuation relation matches with available observations. The Koyna earthquake of 11th December 1967 is still the only large magnitude event in south India for which instrumental strong motion records are available. The actual response spectrum of the horizontal component is compared with the estimated spectrum in Figure 15. It is seen from this figure the instrumental spectrum compares well with the mean spectrum estimated using the frequency dependent attenuation coefficients of Table 3. The sample fluctuations inevitably present in any single record are within the sigma band.

8. Summary

Ground motion data of thirteen earthquakes which occurred in the east coast region of India has been used for calibrating the finite source seismological model. The geometrical attenuation has been taken as that reported by Singh *et al* (1999) for Indian Shield. The kappa factor, which characterizes the near-surface attenuation, has been found out from the slope of the log Fourier amplitude spectra at high frequencies. The kappa factors derived from vertical components are higher than compared to that obtained from horizontal components at all the six stations. Since there is no site specific information regarding the shear wave velocity and density profiles at any ground motion station in the IGCAR array, the site amplification functions are found from the ratio of horizontal and vertical Fourier spectra separately at all the six stations. Since several broad band stations have recorded more than one event, the mean site amplification has been computed at each station. The regional quality factor has been found for east coast region from the local events. The obtained quality factor indicates that the tectonic activity is moderate in the east coast region. After determining the path and site parameters, the stress drop of the seven events has been found out by minimizing the mean square error between the observed and simulated horizontal Fourier spectra. The efficacy of the obtained path and source parameters in simulating ground motion has also been demonstrated in figure 8 by comparing the computed response spectra with the recorded data.

New ground motion relations for spectral acceleration have been developed for east coast region in this article. To derive these equations, finite source seismological model is used to simulate samples of ground motion. The input parameters in the seismological model such as stress drop, depth of the fault, radiation coefficient, pulsing percentage and dip are treated as random variables. The variability of these model parameters are taken from the recorded data and geology. A synthetic database is simulated for east coast region of India. A total of 72,000 ground motion samples have been simulated from 1440 artificial earthquakes covering all ranges of magnitude and distances. These synthetic ground motion samples are further used to derive the empirical equations for east coast region. The developed equations are valid for A-type rock sites in east coast region of India. The proposed attenuation relation has been validated with the strong motion data of 1967 Koyna earthquake. From Figures 15, it is seen that the proposed ground motion relation gives credible values of spectral accelerations for east coast region. These equations can be used by engineers for obtaining the design response spectrum. The results obtained from the present study will be of varied use in seismic analysis and design of infrastructural facilities in east coast region of India. The obtained Quality factor, site amplification and stress drop can be used in the stochastic source seismological model to simulate site-specific acceleration time histories during strong earthquakes in the east coast region.

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